

۲. آزمون عالی نورس

$$f(x) = \begin{cases} (a+1)(x+2) & ; x > 1 \\ 3a+2x & ; x \leq 1 \end{cases}$$

$$2f(\omega) = f(-2) + a$$

$$2((a+1)(\omega+2)) = 3a + \underbrace{(2)(x-2)}_{-2} + a$$

$$(2a+2) \times 2 = 4a-2$$

$$4a+4 = 4a-2$$

$$4a = -2 - 4 = -6 \rightarrow a = \frac{-6}{4} = \frac{-3}{2}$$

$$a = \frac{-18}{10} = \frac{-9}{5}$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$$

$$f(2-\sqrt{3}) + f(2+\sqrt{3})$$

$$\frac{1}{\sqrt{2-\sqrt{3}}} \times \frac{\sqrt{2+\sqrt{3}}}{\sqrt{2+\sqrt{3}}} = \frac{\sqrt{2+\sqrt{3}}}{1}$$

$$f(2-\sqrt{3}) = \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 \Rightarrow \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 + \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2$$

$$f(2+\sqrt{3}) = \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2$$

$$2(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}) + 4 = 2 \times \sqrt{4} + 4 = 2 \times 2 + 4 = 8$$

جواب آخر

$$A^2 = 2\sqrt{3} + 2 + \sqrt{3} + 2(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}) = 4 + 2 = 6 \rightarrow A = \sqrt{6}$$

$$(2f(x) - 3f(x)) = (2x^2 - x) \times 2$$

$$\begin{cases} 4f(x) - 9f(-x) = 12x^2 - 2x \\ 9f(-x) - 9f(x) = 12x^2 + 3x \end{cases}$$

$$\text{اگر } x = -x \rightarrow (2f(-x) - 3f(x)) = (2x^2 + x) \times 2$$

$$-5f(x) = 10x^2 + x$$

$$f(x) = \frac{10x^2 + x}{-5} = \frac{-10x^2 - x}{5} = \frac{-2x^2 - \frac{x}{5}}{1}$$

$$(x+2)f(x) - 3xf(x+2) = 2x^2 - mx + 3m-1$$

$$f(0) = ?$$

$$x=0 \rightarrow 2f(0) = 3x(0)f(2) = 3m-1$$

$$\rightarrow 2f(0) = 3m-1$$

$$f(0) = \frac{3m-1}{2}$$

$$\begin{aligned} 4f(0) &= 5m+15 \\ (2f(0)) &= (3m-1) \times 2 \end{aligned}$$

$$5m+15 = 6m-2$$

$$3m = -17 \quad m = \frac{-17}{3} = \frac{-9}{2}$$

$$x = -2 \rightarrow 4f(0) = 2x^2 + 2m + 3m-1 = 5m+15$$

$$f(0) = \frac{3x(\frac{1}{2})-1}{2} = \frac{\frac{3}{2}-1}{2} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x}$$

$$f(-1) = ? \rightarrow 3x(-1) - 9 = -9$$

$$\left(\frac{ax+b}{x} + \frac{a}{x} + b\right) \times x = \left(\frac{3x^2 - 12x + 3}{x}\right) \times x$$

$$ax^2 + bx + a + bx$$

$$ax^2 + 2b(x) + a = 3x^2 - 12x + 3$$

$$\Rightarrow a=3, \quad 2b=-12 \Rightarrow b=-6 \Rightarrow f(x) = 3x-6$$

$$\begin{cases} f(x) = (x-1)\sqrt{1-x} \\ g(x) = -\sqrt{(1-x)^3} \end{cases}$$

$$\hookrightarrow g(x) = -(1-x)\sqrt{1-x}$$

$$Df = (-\infty, 1]$$

$$Dg = (-\infty, 1]$$

$$1-x \geq 0 \quad 1 \geq x$$

$$(1-x)^3 \geq 0$$

بر د آن ها یکی است اگر عدد جای x بگذاریم یک جواب می دهیم

$$\Rightarrow f(x) = g(x) \text{ این تابع مساوی اند}$$

$$f(x) = x\sqrt{x}$$

$$x \geq 0$$

$$Df = [0, +\infty)$$

$$g(x) = \sqrt{x^3} = x\sqrt{x}$$

$$x^3 \geq 0 \rightarrow x \geq 0$$

$$Dg = [0, +\infty)$$

$$\hookrightarrow f(x) = g(x)$$

مساوی اند

اگر عدد هم جای x بگذاریم مثل هم جواب می دهیم

$$f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor]$$

$$x \geq 0$$

$$Df = [0, +\infty)$$

$$0 \leq \sqrt{x} - \lfloor \sqrt{x} \rfloor < 1 \Rightarrow [\sqrt{x} - \lfloor \sqrt{x} \rfloor] = \text{صفر}$$

$$g(x) = \sqrt{x - |x|}$$

$$x - |x| \geq 0$$

$$x \geq |x| \Rightarrow x \geq 0$$

$$Dg = [0, +\infty)$$

$$\Rightarrow g(x) = 0$$

دامنه های یکسان است و جواب هر تابع یکی است

$$\Rightarrow f(x) = g(x)$$

$$f = \{(1, 2), (2, 3), (3, 1), (0, 0)\}, \quad g = \{(1, -1), (2, 3), (3, 5), (0, 2), (\sqrt{5}, 0)\}$$

$$\text{الف) } \left(\frac{f+g}{f-g}\right)(1) = \frac{f(1)+g(1)}{f(1)-g(1)} = \frac{2+1}{2-1} = \frac{3}{1} = 3$$

$$\text{ب) } \left(\frac{f+g}{f-g}\right)(2) = \frac{f(2)+g(2)}{f(2)-g(2)} = \frac{3+3}{3-3} = \text{تعریف نشده}$$

$$\text{ج) } f(0)g(-1) = 0 \times 1 = \text{صفر}$$

$$f\{(2, 1), (-1, 4), (0, 5), (-2, 2), (3, -1)\}$$

$$\begin{aligned} 3x-1=2 &\rightarrow x=1 & 3x-1=-1 &\rightarrow x=0 \\ 3x-1=4 &\rightarrow x=\frac{5}{3} & 3x-1=2 &\rightarrow x=\frac{3}{3}=1 \end{aligned}$$

$$\text{الف) } g(x) = f(3x-1) \rightarrow g = \{(1, 1), (5, 4), (\frac{1}{3}, 5), (\frac{4}{3}, 2), (\frac{2}{3}, -1)\}$$

$$\text{ب) } h(x) = f(|x|) \rightarrow h = \{(0, 5), (1, 4), (-1, 4), (3, -1), (-3, -1)\}$$

$$|x| = 0, 2, 3 \Rightarrow x \text{ تان } 0, \pm 2, \pm 3$$