

① $r f(x) = f(-x) + a$
 $f(n) = \begin{cases} (a+1)(n+r) & n \geq 1 \\ ra + rx & n \leq 1 \end{cases} \rightarrow (\forall a+r) \times r = ra - 1 + a$
 $a = ? \rightarrow 1 \cdot a = -1 \rightarrow \boxed{a = -1}$ $\div$

② $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + r$ / $f(x-\sqrt{x}) + f(x+\sqrt{x}) = ?$ $\begin{cases} a = \sqrt{x-\sqrt{x}} \\ b = \sqrt{x+\sqrt{x}} \end{cases} \rightarrow a+b=1 \rightarrow \frac{1}{a}=b$
 $a + \frac{1}{a} + r \rightarrow a+b+r$
 $b + \frac{1}{b} + r \rightarrow a+b+r$
 $\Rightarrow ra + rb + 2 \rightarrow a+b = \sqrt{x-\sqrt{x}} + \sqrt{x+\sqrt{x}}$
 $(a+b)^2 = a^2 + b^2 + 2ab$
 $(\sqrt{x-\sqrt{x}} + \sqrt{x+\sqrt{x}})^2 = x - \sqrt{x} + x + \sqrt{x} + 2\sqrt{(x-\sqrt{x})(x+\sqrt{x})}$
 $\Rightarrow r(\sqrt{4}) + 2 \rightarrow \sqrt{4} + 2$

③ $f(n) = ?$ / $r f(n) - r f(-n) = \varepsilon n^2 - x \xrightarrow{\times r} \varepsilon f(n) - r f(-n) = \varepsilon n^2 - rx$
 $x, -x \rightarrow r f(-n) - r f(n) = \varepsilon n^2 + x \xrightarrow{\times r} r f(-n) - r f(n) = \varepsilon n^2 + rx$
 $\rightarrow -2 f(n) = \varepsilon n^2 + x$
 $\boxed{f(n) = -\frac{\varepsilon n^2}{2} - \frac{x}{2}}$ $\div$

④ $(n+r) f(n) - r n f(n+r) = \varepsilon n^2 - m x + r m - 1$ / $f(1) = ?$

$x=0 \rightarrow r f(1) = r m - 1 \xrightarrow{\times r} 4 f(1) = 4 m - 2$
 $x=-r \rightarrow 4 f(1) = 14 + r m + r m - 1 \rightarrow 4 f(1) = 13 + 2 r m$
 $\rightarrow 4 m - 2 = 13 + 2 r m$
 $\varepsilon m = 1 \rightarrow \boxed{m = 1, 2} \rightarrow f(1) = \frac{r m - 1}{r} = \frac{2r - 1}{r}$ $\div$

⑤ $f(n) + f(\frac{1}{n}) = \frac{\varepsilon n^2 - \varepsilon n + r}{n}$ / $f(-1) = ? \xrightarrow{\frac{1}{-1}} -a+b = ?$

$x=-1 \rightarrow f(-1) + f(-1) = \frac{r + r + r}{-1} \rightarrow 2 f(-1) = -1 \rightarrow \boxed{f(-1) = -\frac{1}{2}}$ $\div$

⑥ $Df = x \leq 1$
 $Dg = x \leq 1 \rightarrow Df = Dg, Rf = Rg \rightarrow f(n) = g(n)$ $\div$

✓ $Df = x_{7,0} \rightarrow Df = Dg \quad , \quad Rf = Rg \rightarrow f(n) = g(n)$ برابری

⋈ $Df = x_{7,0} \rightarrow$
 $Dg = x_{7,0}$
 استند ۲ عبارت زیر در بیان هر دو قراداد است (این مفید است)
 هم‌نوع برابری و هم‌نوع هم
 عبارت زیر در بیان هم‌نوع هم!

⑨ الف) $\frac{f(1)+g(1)}{f(1)-g(1)} \rightarrow \frac{2+1}{2-1} = \frac{3}{1} = 3$ ب) $\frac{f(2)+g(2)}{f(2)-g(2)} \rightarrow \frac{4+2}{4-2} = \frac{6}{2} = 3$

ج) $f(1) \times g(-1) \rightarrow 3 \times 1 = 3$

⑩ الف) $g(n) = f(x_{n-1}) \rightarrow$

$$\begin{cases} x_{n-1} = 2 \rightarrow n = 1 \rightarrow g(1) = 1 \\ n = -1 \rightarrow n = 0 \rightarrow g(0) = 2 \\ n = 0 \rightarrow n = \frac{1}{2} \rightarrow g(\frac{1}{2}) = 2 \\ n = -2 \rightarrow n = -\frac{1}{2} \rightarrow g(-\frac{1}{2}) = 2 \\ n = 3 \rightarrow n = \frac{2}{3} \rightarrow g(\frac{2}{3}) = -1 \end{cases}$$

$g = \left\{ (1,1), (0,2), (\frac{1}{2},2), (-\frac{1}{2},2), (\frac{2}{3},-1) \right\}$

ب) $h(n) = f(|n|)$

$|n| = 2 \rightarrow n = \begin{cases} +2 \rightarrow y = 1 \rightarrow h(2) = 1 \\ -2 \rightarrow y = 1 \rightarrow h(-2) = 1 \end{cases}$

~~$|n| = 1$~~

$|n| = 0 \rightarrow n = 0 \rightarrow y = 2 \rightarrow h(0) = 2$

~~$|n| = -2$~~

$|n| = 3 \rightarrow n = \begin{cases} +3 \rightarrow y = -1 \rightarrow h(3) = -1 \\ -3 \rightarrow y = -1 \rightarrow h(-3) = -1 \end{cases}$

$h(n) = \left\{ (2,1), (-2,1), (3,-1), (-3,-1), (0,2) \right\}$