

$$2f(\omega) = 2(a+1)(\omega+2) = 14(a+1) = 14a+14$$

$$f(-2)+a = 3a-4+a = 4a-4$$

$$2f(\omega) = f(-2)+a \rightarrow 14a+14 = 4a-4 \rightarrow 10a = -18 \rightarrow a = \frac{-18}{10} = -\frac{9}{5}$$

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$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \quad ; \quad f(2-\sqrt{3}) + f(2+\sqrt{3}) = \sqrt{9} + \sqrt{9} + 4 = 4 + 2\sqrt{9}$$

$$\rightarrow f(2-\sqrt{3}) = \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 \quad \left\{ \begin{array}{l} A = \sqrt{2-\sqrt{3}}, B = \sqrt{2+\sqrt{3}} \\ AB = \sqrt{(2-\sqrt{3})(2+\sqrt{3})} = \sqrt{2^2 - (\sqrt{3})^2} = \sqrt{4-3} = 1 \\ (a-b)(a+b) = a^2 - b^2 \end{array} \right.$$

$$\rightarrow f(2+\sqrt{3}) = \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2$$

$$(A+B)^2 = A^2 + B^2 + 2AB \rightarrow 2 - \sqrt{3} + 2 + \sqrt{3} + 2 = 6 \rightarrow A+B = \sqrt{9} \quad ; \quad \frac{1}{A} + \frac{1}{B} = \frac{A+B}{AB} = \frac{A+B}{1} = A+B = \sqrt{9}$$

جواب
نهایی

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$$\left. \begin{array}{l} 2f(x) - 3f(-x) = 4x^2 - 2x \quad \times x^2 \rightarrow 4f(x) - 9f(-x) = 14x^3 - 2x \\ 2f(-x) - 3f(x) = 4x^2 + 2x \quad \times x^3 \rightarrow -9f(x) + 4f(-x) = 14x^3 + 3x \end{array} \right\}$$

$$\rightarrow -5f(x) = 10x^3 + x \rightarrow f(x) = -2x^3 - \frac{x}{5}$$

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$$(x+2)f(x) - 2xf(x+2) = 4x^2 - mx + m - 1$$

$$\left. \begin{array}{l} x \rightarrow 2f(0) - 0 = 2m - 1 \rightarrow f(0) = \frac{2m-1}{2} \\ x \rightarrow 4f(0) = 12 + 2m \rightarrow f(0) = \frac{12+m}{4} \end{array} \right\} \frac{2m-1}{2} = \frac{12+m}{4} \times 4 \rightarrow 4m - 2 = 12 + m \rightarrow 3m = 14$$

$$f(0) = \frac{2m-1}{2} = \frac{2(\frac{14}{3})-1}{2} = \frac{15}{3} = 5$$

$$\left\{ \begin{array}{l} 3m = 14 \\ m = \frac{14}{3} \end{array} \right.$$

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$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^3 - 12x + 3}{x} \div x \rightarrow 3x - 12 + \frac{3}{x}$$

$$\downarrow \quad \downarrow$$

$$ax+b \quad \frac{a}{x}+b$$

$$ax+b + \frac{a}{x}+b = ax + \frac{a}{x} + 2b \quad \left\{ \begin{array}{l} ax + \frac{a}{x} + 2b = 3x - 12 + \frac{3}{x} \rightarrow a=3 \\ b=-9 \end{array} \right.$$

$$f(x) = 3x - 9 \rightarrow f(-1) = -3 - 9 = -12$$

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$$f(x) = (x-1)\sqrt{1-x} \rightarrow 1-x \geq 0 \rightarrow x \leq 1$$

$$g(x) = -\sqrt{(1-x)^3} \rightarrow (1-x)^3 \geq 0 \rightarrow 1-x \geq 0 \rightarrow x \leq 1$$

$$D_f = (-\infty, 1]$$

$$D_g = (-\infty, 1]$$

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$$f(x) = x\sqrt{x} \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty)$$

$$g(x) = \sqrt{x^3} \rightarrow x^3 \geq 0 \rightarrow x \geq 0 \rightarrow D_g = [0, +\infty)$$

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$$f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty)$$

$$g(x) = \sqrt{x - |x|} \rightarrow x - |x| \geq 0 \rightarrow x \geq |x| \rightarrow D = [0, +\infty)$$

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$$f = \{(1, 2), (2, 3), (3, 1), (0, 0)\}, g = \{(-1, 1), (1, 1), (2, 3), (3, 5), (0, 2), (\sqrt{7}, 1)\}$$

$$\text{الف) } \left(\frac{f+g}{f-g}\right)(1) \rightarrow f(1)=2, g(1)=1 \rightarrow \frac{2+1}{2-1} = 3$$

$$\text{ب) } \left(\frac{f+g}{f-g}\right)(2) \rightarrow f(2)=3, g(2)=3 \rightarrow \frac{3+3}{3-3} = \frac{6}{0} \text{ تعریف نشده}$$

$$\text{ج) } f(0)g(-1) \rightarrow f(0)=0, g(-1)=1 \rightarrow 0 \times 1 = 0$$

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$$f(x) = \{(2, 1), (-1, 4), (0, 5), (-2, 2), (3, -1)\}$$

$$\text{الف) } g(x) = f(3x-1) \rightarrow 3x-1 = -2 \rightarrow x = -\frac{1}{3} \mid 3x-1 = -1 \rightarrow x = 0 \mid 3x-1 = 0 \rightarrow x = \frac{1}{3}$$

$$3x-1 = 2 \rightarrow x = 1 \mid 3x-1 = 3 \rightarrow x = \frac{4}{3} \rightarrow g = \left\{(-\frac{1}{3}, 2), (0, 4), (1, 1), (\frac{1}{3}, 5), (\frac{4}{3}, -1)\right\}$$

$$\text{ب) } h(x) = f(|x|) \rightarrow |x| = 0 \rightarrow x = 0 \mid |x| = 2 \rightarrow x = \pm 2 \mid |x| = 3 \rightarrow x = \pm 3$$

$$|x| \geq 0 \mid h = \{(0, 5), (2, 1), (-2, 1), (3, -1), (-3, -1)\}$$

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