

نام و نام خانوادگی کلاس شماره پاسخنامه تشریحی تکلیف شماره ۲۰

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$$f(x) = \begin{cases} (a+1)(x+2) & ; x > 1 \xrightarrow{x=5} (a+1)(5) \rightarrow 5a+5 = f(5) \\ 3a+2x & ; x \leq 1 \xrightarrow{x=-2} 3a+2(-2) \rightarrow 3a-4 = f(-2) \end{cases}$$

$$\star 2f(5) = f(-2) + a \quad ? \quad \xrightarrow{15a+15} \xrightarrow{3a} 2(5a+5) = 3a-4+a \rightarrow 10a = -14 \rightarrow a = -1.4 \quad \text{ج. ۲}$$

$$a = ? \rightarrow -1.4 \quad / \quad \left(-\frac{9}{5} \right)$$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \rightarrow \frac{x+1+2\sqrt{x}}{\sqrt{x}} \rightarrow \frac{(\sqrt{x}+1)^2}{\sqrt{x}}$$

★ داخل بولتنه: $(2-\sqrt{3}+2+\sqrt{3}+2) = 6$
بجز جواب $(\sqrt{6})$

$$\star f(2-\sqrt{3}) + f(2+\sqrt{3}) = ? \quad \xrightarrow{\text{نویسنده}} \sqrt{2-\sqrt{3}} + \frac{1}{\sqrt{2-\sqrt{3}}} + 2 + \sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}} + 2$$

$$\Rightarrow \sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}} + \sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}} + 4$$

$$\star 2\sqrt{2-\sqrt{3}} + 2\sqrt{2+\sqrt{3}} = 2(\sqrt{2-\sqrt{3}} + \sqrt{2+\sqrt{3}}) + 4 \quad \xrightarrow{2(\sqrt{6})+4} \xrightarrow{2\sqrt{6}+4} \text{ج. ۲}$$

$$\begin{aligned} 2 \times & \int 2f(x) - 3f(-x) = f(x^2 - x) \\ 3 \times & \int 3f(-x) - 2f(x) = f(x^2 + x) \end{aligned}$$

$$\Rightarrow \frac{-5f(x)}{f(x) - 9f(x)} = \frac{12x^2 - 2x}{2(f(x^2 - x) + 3(f(x^2 + x)))} \rightarrow -5f(x) = 10x^2 + x$$

$$f(x) = -2x^2 - \frac{x}{5} \quad \text{ج. ۳}$$

$$f(x) = ?$$

$$(x+2)f(x) - 3xf(x+2) = f(x^2 - mx + 3m - 1) \quad \xrightarrow{x=0} 2f(0) = 3m-1 \rightarrow f(0) = \frac{15m-0.5}{2}$$

$$\xrightarrow{x=-2} 2f(0) = 12 + \frac{3m+3m-1}{15} \rightarrow f(0) = \dots$$

$$\star 9m - 3 = 15 + 3m \rightarrow 6m = 18 \rightarrow m = \frac{18}{6} = 3$$

$$\star f(0) = \frac{3}{5} \times \frac{18^9}{5} - \frac{1 \times 2}{5 \times 2} \rightarrow \frac{18^9}{5} \quad \text{ج. ۲}$$

$$\star f(-1) + f(-1) = \frac{18}{-1} \rightarrow 2f(-1) = -18 \rightarrow f(-1) = -9 \quad \text{ج. ۲} \quad \boxed{ax+b}$$

$$\star 3(-1) - 6 = -9 \quad \text{ج. ۲} \quad \boxed{3x-6}$$

$$\star f(x) + f\left(\frac{1}{x}\right) = \frac{ax}{x} + \frac{bx}{x} \rightarrow \frac{a}{x}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$ax+b : \text{نسبت} \quad \frac{a}{x} + b : \text{نسبت} \quad \rightarrow ax+b + \frac{a}{x} + b = \frac{3x^2 - 12x + 3}{x} \rightarrow 2b$$

$$\begin{cases} 2b = -12 \rightarrow b = -6 \checkmark \\ 3x = ax \rightarrow a = 3 / \frac{3}{x} = \frac{a}{x} \rightarrow a = 3 \checkmark \end{cases}$$

$$\left\{ \begin{array}{l} f(x) = (2x-1)\sqrt{1-x} \rightarrow x \leq 1 \\ g(x) = -\sqrt{(1-x)^3} \end{array} \right. \quad \left. \begin{array}{l} \begin{array}{c} + \quad - \\ | \quad - \end{array} \rightarrow (-\infty, 1] \\ \begin{array}{c} + \quad - \\ | \quad - \end{array} \rightarrow (-\infty, 1] \end{array} \right\} \quad \begin{array}{l} f(x) = g(x) \\ \text{مسایه ای دارند} \\ \text{جای ندارند} \end{array}$$

دارای دامنه یکسان

$$\left\{ \begin{array}{l} f(x) = x\sqrt{x} \rightarrow [0, +\infty) \\ g(x) = \sqrt{x^3} \rightarrow \mathbb{R}^+ \cup \{0\} \rightarrow [0, +\infty) \end{array} \right. \quad \left. \begin{array}{l} f(x) = g(x) \\ \text{مسایه ای دارند} \\ \text{جای ندارند} \end{array} \right.$$

$$\left\{ \begin{array}{l} f(x) = [\sqrt{x} - \lfloor \sqrt{x} \rfloor] \rightarrow [0, 1) \\ g(x) = \sqrt{x - |x|} \rightarrow [0, +\infty) \end{array} \right. \quad \left. \begin{array}{l} f(x) = g(x) \\ \text{مسایه ای دارند} \end{array} \right.$$

الف) $\left[\frac{p+q}{p-q} \right] (1) \rightarrow \frac{2+1}{2-1} = 3$ ر.ع.
 ب) $\left[\frac{p+q}{p-q} \right] (2) \rightarrow \frac{3+3}{3-3} = \text{تعیین}$ ر.ع.
 ج) $f(0)g(-1) = 0$ ر.ع.

الف) $g(x) = f(3x-1) \rightarrow g(x) = \left\{ (1, 1), (0, 4), \left(\frac{1}{3}, 5\right), \left(-\frac{1}{3}, 2\right), \left(\frac{4}{3}, -1\right) \right\}$ ر.ع.
 $\hookrightarrow 3x-1=2 \rightarrow x=1 \quad 3x-1=-1 \rightarrow x=0 \quad 3x-1=0 \rightarrow x=\frac{1}{3} \quad 3x-1=-2 \rightarrow x=-\frac{1}{3} \quad 3x-1=3 \rightarrow x=\frac{4}{3}$
 ب) $h(x) = f(|x|) \rightarrow h(x) = \left\{ (0, 5), (2, 1), (3, -1), (-3, -1), (-2, 1) \right\}$ ر.ع.
 $f(|x|) \rightarrow x = 2, 0, 3 \quad |x|=2 \quad |x|=3$
 ج) $f(x)g(x) = 0$ ر.ع.