

$$f(x) = (x+1)^2 = x^2 + 2x + 1 \rightarrow f'(x) = 2x + 2$$

$$f(-1) = 0 \rightarrow f'(-1) = 0$$

$$\Rightarrow 2x + 2 = 0 \Rightarrow 2x = -2 \Rightarrow x = -1$$

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$$f(x+\sqrt{x}) = \sqrt{x+\sqrt{x}} + \frac{1}{\sqrt{x+\sqrt{x}}} + x$$

$$f(x-\sqrt{x}) = \sqrt{x-\sqrt{x}} + \frac{1}{\sqrt{x-\sqrt{x}}} + x$$

$$x+\sqrt{x} - (x-\sqrt{x}) = 2\sqrt{x}$$

$$\frac{1}{x+\sqrt{x}} - \frac{1}{x-\sqrt{x}} = \frac{1}{\sqrt{x+\sqrt{x}}} - \frac{1}{\sqrt{x-\sqrt{x}}}$$

$$\rightarrow \frac{1}{\sqrt{x+\sqrt{x}}} = \frac{1}{\sqrt{x-\sqrt{x}}}$$

$$\frac{(\sqrt{x+\sqrt{x}} + \sqrt{x-\sqrt{x}} + x) + (\sqrt{x+\sqrt{x}} + \sqrt{x-\sqrt{x}} + x)}{2}$$

$$x^2 (\sqrt{x+\sqrt{x}} + \sqrt{x-\sqrt{x}})$$

$$x^2 (x + \sqrt{x}) = x^2 + x\sqrt{x} \quad n = \sqrt{x}$$

$$f(x+\sqrt{x}) + f(x-\sqrt{x}) = 2x + 2\sqrt{x}$$

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$$f'(x) - f'(-x) = 2x^2 - 2(-x)^2 = 2x^2 - 2x^2 = 0$$

$$f'(-x) - f'(x) = 2(-x)^2 - 2(x)^2 = 2x^2 - 2x^2 = 0$$

$$f'(x) - 4f(-x) = 2x^2 - 4(-x)^2 = 2x^2 - 4x^2 = -2x^2$$

$$-4f'(x) + 4f(-x) = -4(2x^2) + 4(-x)^2 = -8x^2 + 4x^2 = -4x^2$$

$$f'(0) = 2m - 1 \rightarrow m = \frac{1 + f'(0)}{2}$$

$$4f'(0) = 14 + 2m + 2m - 1$$

$$4f'(0) = 14 + \frac{4 + 1 \cdot f'(0)}{2} \rightarrow 14f'(0) = 14 + 4 + 1 \cdot f'(0) \rightarrow 14f'(0) = 18 + f'(0) \rightarrow f'(0) = \frac{18}{13}$$

3

$$f(-1) + f(-1) = \frac{1+1+1}{-1} = -3 \quad f(-1) = -1.5$$

$$f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b = \frac{ax^2 + 2bx + a}{x} \rightarrow a = 1, b = -4$$

$$f(-1) = 1(-1) + (-4) = -5$$

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$$\begin{cases} f(x) = (x-1)\sqrt{1-x} \rightarrow 1-x \geq 0 \rightarrow -1 \geq x \Rightarrow D_f = (-\infty, 1] \rightarrow \sqrt{1-x}(x-1)^2 \\ g(x) = \sqrt{(1-x)^3} \rightarrow (1-x)^3 \geq 0 \rightarrow 1-x \geq 0 \rightarrow (-\infty, 1] \quad D_f = D_g \\ f(x) = -(1-x)\sqrt{1-x} = -\sqrt{(1-x)^3} = -g(x) \\ \Rightarrow f = -g \end{cases}$$

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$$\begin{cases} f(x) = x\sqrt{x} \rightarrow x \geq 0 \Rightarrow D_f = [0, +\infty) \\ g(x) = \sqrt{x^3} \rightarrow x^3 \geq 0 \Rightarrow x \geq 0 \Rightarrow D_g = [0, +\infty) \\ \Rightarrow D_f = D_g \\ \forall x \geq 0 \rightarrow f(x) = x\sqrt{x} = \sqrt{x^2 \cdot x} = \sqrt{x^3} = g(x) \\ \Rightarrow f = g \end{cases}$$

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$$\begin{cases} f(x) = [\sqrt{x} \cdot \sqrt{x}] \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty) \\ g(x) = \sqrt{x \cdot x} \rightarrow x - |x| \geq 0 \rightarrow D_g = \mathbb{R} \\ D_f \neq D_g \\ \Rightarrow f \neq g \end{cases}$$

! ملاحظة: $x - |x| \geq 0 \rightarrow x \geq |x| \rightarrow x \geq 0$

1, 2, 3

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$$\begin{aligned} \frac{f(1) + g(1)}{f(1) - g(1)} &= \frac{1+1}{1-1} = \frac{2}{0} = \text{غير معرف} \\ \frac{f(-1) + g(-1)}{f(-1) - g(-1)} &= \frac{1+1}{1-1} = \frac{2}{0} = \text{غير معرف} \\ f(0) \times g(-1) &= 0 \times 1 = 0 \end{aligned}$$

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$$\begin{aligned} 1^2 x - 1 = 1 &\Rightarrow 1^2 x = 2 \Rightarrow x = \frac{2}{1} \rightarrow \left(\frac{2}{1}, -1\right) \\ 1^2 x - 1 = -2 &\Rightarrow 1^2 x = -1 \Rightarrow x = -1 \rightarrow \left(-\frac{1}{1}, 2\right) \\ 1^2 x - 1 = 0 &\Rightarrow 1^2 x = 1 \Rightarrow x = \frac{1}{1} \rightarrow \left(\frac{1}{1}, 0\right) \\ 1^2 x - 1 = -1 &\Rightarrow 1^2 x = 0 \Rightarrow x = 0 \rightarrow (0, 1) \\ 1^2 x - 1 = 1 &\Rightarrow 1^2 x = 2 \Rightarrow x = 1 \rightarrow (1, 1) \\ g &= \left\{ \left(\frac{2}{1}, -1\right), \left(-\frac{1}{1}, 2\right), \left(\frac{1}{1}, 0\right), (0, 1), (1, 1) \right\} \end{aligned}$$

$$\begin{aligned} |x| = 1 &\Rightarrow x = 1 \Rightarrow (1, -1) \\ &\quad x = -1 \Rightarrow (-1, 1) \\ |x| = -1 &\Rightarrow \text{غير ممكن} \\ |x| = 0 &\Rightarrow x = 0 \Rightarrow (0, 0) \\ |x| = 1 &\Rightarrow \text{غير ممكن} \\ |x| = 1 &\Rightarrow x = -1, x = 1 \Rightarrow (1, 1), (-1, 1) \\ h &= \{(1, -1), (-1, -1), (0, 0), (1, 1), (-1, 1)\} \end{aligned}$$

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$$f(n) = [\sqrt{n} - [\sqrt{n}]] \rightarrow \sqrt{n} \rightarrow n \geq 0 \rightarrow D_f = [0 + \infty)$$

$$g(n) = \sqrt{n-1} \rightarrow n-1 \geq 0 \rightarrow D_g = [0 + \infty)$$

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$$f(n) = [\sqrt{n}] - [\sqrt{n}] = 0 \quad \text{عنصمیم از برای آن می آید بیرون ت}$$

$$g(n) = 0 \quad \text{نهایت صغری است!} \rightarrow n-1 \geq 0 \rightarrow n-1 = n-1 \geq 0 \rightarrow n \geq 1$$