

$$f(x) = (a+1)x = ax + x \rightarrow f'(x) = a + 1$$

$$f(-x) = ax - x \rightarrow f'(-x) + a = a - 1$$

$$\Rightarrow a + 1 = a - 1 \Rightarrow 1 = -1 \Rightarrow a = -1$$

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$$f(x + \sqrt{x}) = \sqrt{x + \sqrt{x}} + \frac{1}{\sqrt{x + \sqrt{x}}} + x$$

$$f(x - \sqrt{x}) = \sqrt{x - \sqrt{x}} + \frac{1}{\sqrt{x - \sqrt{x}}} + x$$

$$x + \sqrt{x} - (x - \sqrt{x}) = 2\sqrt{x} = 1$$

$$\frac{1}{x + \sqrt{x}} = x - \sqrt{x} \Rightarrow \frac{1}{\sqrt{x + \sqrt{x}}} = \sqrt{x - \sqrt{x}}$$

$$\rightarrow \frac{1}{\sqrt{x - \sqrt{x}}} = \sqrt{x + \sqrt{x}}$$

$$\frac{(\sqrt{x + \sqrt{x}} + \sqrt{x - \sqrt{x}} + x) + (\sqrt{x + \sqrt{x}} + \sqrt{x - \sqrt{x}} + x)}{n}$$

$$x^c (\sqrt{x + \sqrt{x}} + \sqrt{x - \sqrt{x}})^c$$

$$x^c = x + x\sqrt{x} = x + x = 2x \quad n = \sqrt{4}$$

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$$f'(x) - f'(-x) = 2x^c$$

$$f'(-x) - f'(x) = 2(-x)^c = -2x^c \Rightarrow x^c = x \Rightarrow c = 1$$

$$f'(x) - 4f'(-x) = 12x^c - 4x$$

$$-4f'(x) + 4f'(-x) = 12x^c + 4x \Rightarrow -4f'(x) = 12x^c + 4x - 4f'(-x)$$

$$\Rightarrow -4f'(x) = 12x^c + 4x - 4(-x^c) = 12x^c + 4x + 4x^c = 16x^c + 4x$$

$$\Rightarrow -4f'(x) = 16x^c + 4x \Rightarrow f'(x) = -4x^c - x$$

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$$f'(0) = 2m - 1 \Rightarrow m' = \frac{1 + f'(0)}{2}$$

$$4f'(0) = 14 + 2m + 2m - 1$$

$$4f'(0) = 14 + \frac{2 + f'(0)}{2} \Rightarrow 14f'(0) = 14 + 2 + f'(0) \Rightarrow 13f'(0) = 16 \Rightarrow f'(0) = \frac{16}{13}$$

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$$f(-1) + f(-1) = \frac{1 + 1 + 1}{-1} = -3 \quad f(-1) = -9$$

$$f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b = \frac{ax^2 + 2bx + a}{x} \Rightarrow a = 1, b = -4$$

$$f(-1) = 1(-1) + (-4) = -5$$

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$$\begin{cases} f(x) = (x-1)\sqrt{1-x} \rightarrow 1-x \geq 0 \rightarrow -1 \geq x \Rightarrow D_f = (-\infty, 1] \rightarrow \sqrt{1-x}(x-1)^2 \\ g(x) = \sqrt{(1-x)^3} \rightarrow (1-x)^3 \geq 0 \rightarrow 1-x \geq 0 \rightarrow (-\infty, 1] \quad D_f = D_g \\ f(x) = -(1-x)\sqrt{1-x} = -\sqrt{(1-x)^3(1-x)} = -\sqrt{(1-x)^5} = g(x) \\ \Rightarrow f = g \end{cases}$$

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$$\begin{cases} f(x) = x\sqrt{x} \rightarrow x \geq 0 \Rightarrow D_f = [0, +\infty) \\ g(x) = \sqrt{x^3} \rightarrow x^3 \geq 0 \Rightarrow x \geq 0 \Rightarrow D_g = [0, +\infty) \Rightarrow D_f = D_g \\ \forall x \geq 0 \rightarrow f(x) = x\sqrt{x} = \sqrt{x^2 \cdot x} = \sqrt{x^3} = g(x) \\ \Rightarrow f = g \end{cases}$$

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$$\begin{cases} f(x) = [\sqrt{x} \cdot \sqrt{x}] \rightarrow x \geq 0 \rightarrow D_f = [0, +\infty) \\ g(x) = \sqrt{x - [x]} \rightarrow x - [x] \geq 0 \rightarrow D_g = \mathbb{R} \\ \Rightarrow D_f \neq D_g \\ \Rightarrow f \neq g \end{cases}$$

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$$\frac{f(1) + g(1)}{f(1) - g(1)} = \frac{1+1}{1-1} = \frac{2}{0} = \infty$$

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$$\frac{f(-1) + g(-1)}{f(-1) - g(-1)} = \frac{1+1}{1-1} = \frac{2}{0} = \infty$$

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$$f(0) \times g(-1) = 0 \times 1 = 0$$

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$$\begin{aligned} 1^2x-1=1 &\Rightarrow 1^2x=2 \Rightarrow x=2 \rightarrow \left(\frac{2}{1}, -1\right) \\ 1^2x-1=-2 &\Rightarrow 1^2x=-1 \Rightarrow x=-1 \rightarrow \left(-\frac{1}{1}, -2\right) \\ 1^2x-1=0 &\Rightarrow 1^2x=1 \Rightarrow x=1 \rightarrow \left(\frac{1}{1}, 0\right) \\ 1^2x-1=-1 &\Rightarrow x=0 \rightarrow (0, -1) \\ 1^2x-1=1 &\Rightarrow 1^2x=2 \Rightarrow x=2 \rightarrow (2, 1) \\ g = \left\{ \left(\frac{2}{1}, -1\right), \left(-\frac{1}{1}, -2\right), \left(\frac{1}{1}, 0\right), (0, -1), (2, 1) \right\} \end{aligned}$$

$$\begin{aligned} |x|=1 &\Rightarrow x=1 \Rightarrow (1, -1) \\ &\quad x=-1 \Rightarrow (-1, -1) \\ |x|=0 &\Rightarrow x=0 \Rightarrow (0, 0) \\ |x|=1 &\Rightarrow x=1 \Rightarrow (1, 1) \\ &\quad x=-1 \Rightarrow (-1, 1) \\ k = \{(1, -1), (-1, -1), (0, 0), (1, 1), (-1, 1)\} \end{aligned}$$

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