

۱۹، ۵ آزمون به سه

۱

$$f(x) = \sqrt{x+4\sqrt{x-1}} \xrightarrow{\text{تبدیل به یک}} \sqrt{x-1+4\sqrt{x-1}+4} = \sqrt{(\sqrt{x-1}+2)^2} = |\sqrt{x-1}+2|$$

$$g(x) = \sqrt{x-4\sqrt{x-1}} \xrightarrow{\text{تبدیل به یک}} \sqrt{x-1-4\sqrt{x-1}+4} = \sqrt{(\sqrt{x-1}-2)^2} = |\sqrt{x-1}-2|$$

$$x > 1 \rightarrow f(x) + g(x) = a + b\sqrt{x-1} + c \quad f(x) + g(x) = \sqrt{x-1} + 2 + \sqrt{x-1} - 2 = 2\sqrt{x-1}$$

$$x > 1 \Rightarrow |\sqrt{x-1} + 2| = 2 \quad \text{چون} \quad 2\sqrt{x-1} = a + b\sqrt{x-1} + c$$

$$x > 1 \Rightarrow |\sqrt{x-1} - 2| = 0 \quad \text{چون} \quad a = 0 \quad c = -2 \quad \frac{a+b}{c} = \frac{0+2}{-2} = -1 \quad \text{پس} \quad \frac{1}{2} \quad \text{پ}$$

۲

$$f(x) = \frac{x+2}{x^2-2x+3} \rightarrow x^2-2x+3 \neq 0 \rightarrow a+b+c=0 \rightarrow x=1 \rightarrow \frac{g}{f} \rightarrow f \neq 0$$

$$g(x) = \left\{ (2, 1), (1, 0), (3, 0), (0, 4) \right\} \quad f(2) = \frac{2+2}{2^2-2 \cdot 2+3} = \frac{4}{-1} = -4 \quad f(0) = \frac{2}{3}$$

$$\frac{g}{f} = \left\{ (2, \frac{1}{-4}), (0, \frac{4}{\frac{2}{3}}) \right\} \rightarrow \frac{g}{f} = \left\{ (2, -\frac{1}{4}), (0, 6) \right\} \quad \text{پ}$$

۳

$$f(x) = \{(1, 2), (3, -1), (4, 2), (-1, 4)\} \quad g(x) = \{(-2, 3), (1, -1), (2, 2), (-1, 0)\}$$

$$\text{الف) } f(x) \rightarrow x \in D_f \rightarrow \left\{ \left(\frac{1}{2}, 2\right), \left(\frac{1}{2}, -1\right), (2, 2), \left(-\frac{1}{2}, 4\right) \right\}$$

$$\text{ب) } f(x) \rightarrow x \in D_f \rightarrow \{(1, 2), (-1, 2), (\sqrt{3}, -1), (-\sqrt{3}, -1), (2, 2)\} \rightarrow x^2 = -1 \quad \text{ریشه حقیقی ندارد}$$

$$\text{ج) } \left\{ (-2, 1), (1, 3), (4, -1), (-1, 1) \right\} \quad \text{دقت!}$$

$$\text{د) } \frac{f}{g} \rightarrow \frac{0}{0} = \{1, 3, -1\} \cdot \left\{ (1, -4), (3, -1), (-1, \frac{11}{4}) \right\} = \{(1, -4), (3, -1)\} \quad \text{پ}$$

۴

$$f(x) = \{(1, 1), (2, 1), (4, 9), (-2, 0)\} \quad h = f + g$$

$$g(x) = \{(0, 1), (2, 3), (-2, 4), (-1, 2), (3, 3)\} \quad h = \{(1, 0), (2, 4), (-2, 4)\}$$

$$\text{دامنه} = \{1, 2, -2\} \quad \text{پ}$$

۵

$$f(x) = \frac{\sqrt{x^3-2x}}{x-2} \quad \text{شرط: } x^3-2x \geq 0 \rightarrow x(x^2-2) \geq 0 \rightarrow x \geq 0, x \leq \sqrt{2}$$

$$g(x) = [x] + [-x]$$

$$D\left(\frac{f}{g+1}\right)(x) \quad \frac{f}{g+1} \rightarrow \frac{f}{g+1} \neq 0 \quad g(x) = \begin{cases} 0 & x \in \mathbb{Z} \\ -1 & x \notin \mathbb{Z} \end{cases}$$

$$\text{شرط منفی: } x \cdot x \neq 0 \rightarrow x \neq 0$$

$$\frac{0}{-1+1} \rightarrow [0, 3]$$

$$Df = [0, 3] - \{2\}$$

$x \in \mathbb{Z} \cap [0, 3] - \{2\} = \{0, 1, 3\} \quad \text{پ}$

$D\left(\frac{f}{g+1}\right)(x) = \{0, 1, 3\}$

$$f(n) = \sqrt{-n^2 + 5n - 2} \rightarrow -n^2 + 5n - 2 \geq 0 \quad -n^2 + 5n - 2 = 0 \rightarrow a_1 + b + c x \quad n=1, n=2 \quad \frac{1}{-b} + \frac{1}{b} = \frac{1}{c/a}$$

$$D = [1, 4]$$

$$g(n) = \{(1, 3), (2, 2), (3, 4), (4, 1)\}$$

$$D = \{1, 2, 3, 4\}$$

$$f(1) = 0 \quad f(2) = \sqrt{-2 + 10 - 2} = 2$$

$$h(n) = \frac{f(n)}{g(n)}$$

$$h(n) = \{(1, 0), (2, 0), (3, \frac{4}{3}), (4, 1)\}$$

الف) $y = n^3 - 3n^2 + 3n - 1 + 1 \rightarrow y = (n-1)^3 + 1 \quad y-1 = (n-1)^3$
 $n-1 = \sqrt[3]{y-1} \rightarrow n = \sqrt[3]{y-1} + 1 \quad R = \mathbb{R}$

$$y = \frac{1}{n^2 - 2n}$$

$$\rightarrow y n^2 - 2y n = 1$$

$$y n^2 - 2y n - 1 = 0 \rightarrow n = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y}$$

$$n = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y}$$

بفرض $y \neq 0$ عبارت y را فرض کنید!

الف) $y = n^2 - 4n + 10 \quad nS = \frac{-b}{2a} = \frac{4}{2} = 2 \quad yS = 4 - 16 + 10 = -2 \quad R = [-2, +\infty)$

ب) $y = -n^2 + 4n + 3 \quad nS = \frac{-b}{2a} = \frac{-4}{-2} = 2 \quad yS = -9 + 16 + 3 = 10 \quad R = (-\infty, 10]$

ج) $y = \sqrt{n^2 - 4n - 5} \quad nS = \frac{-b}{2a} = \frac{4}{2} = 2 \quad yS = 4 - 8 - 5 = -9 \rightarrow [-9, +\infty) \rightarrow \sqrt{[-9, +\infty)} = [-3, +\infty) = R$

د) $y = \sqrt{-n^2 + 4n} \quad nS = \frac{-b}{2a} = \frac{-4}{-2} = 2 \quad yS = -9 + 16 = 7 \rightarrow [0, 7] \rightarrow R = [0, 7]$

الف) $y = n^2 - 8n + 2n + 1 \quad R = \mathbb{R}$

ب) $y = n^2 - 4n^2 + 3n + 2 \quad R = \mathbb{R}$

ج) $y = \sqrt{n^2 - 2n^2 + 4n + 1} \rightarrow R = \sqrt{1R} \rightarrow R = [1, +\infty)$

د) $y = (n^2 - 4n^2 + (n+1)^2)^2 \rightarrow R = (\mathbb{R})^2 \rightarrow R = [0, +\infty)$

الف) $y = \frac{n+1}{n-2} \quad R = \mathbb{R} - \{2\} \quad \left\{ \frac{n+1}{n-2} = y \Rightarrow n = \frac{y+1}{y-2} \right. \quad n = \frac{y+1}{y-2} \rightarrow y \neq 2$

ب) $y = \frac{n+1}{n+1} \quad R = \mathbb{R} - \{2\} \quad \left\{ \frac{n+1}{n+1} = y \Rightarrow n = \frac{y-1}{y-2} \right. \quad n = \frac{y-1}{y-2} \rightarrow y \neq 2$

$$f(2n) = \left\{ \left(\frac{1}{2}, 2\right) \left(\frac{\sqrt{2}}{2}, -1\right) (2, 2) \left(-\frac{1}{2}, 4\right) \right\} \quad \text{ن های } f \text{ باید حقیق برابر شوند!} \quad (3)$$

$$f(2) = \left\{ (1, 2) (-1, 2) (\sqrt{2}, -1) (-\sqrt{2}, -1) (2, 2) (-2, 2) \right\} \quad \text{هر کدام از } n \text{ های } f \text{ را برابر } n^2 \text{ قرار دهیم!}$$

$$g^2(n) + 1 = \left\{ (-2, 19) (1, 3) (3, 9) (-1, 1) \right\} \quad \text{این عمل روی } n \text{ های و اعمال می شود:}$$

$$\frac{2f}{g} = \left\{ \left(1, \frac{2 \times 2}{-1}\right) \left(3, \frac{2 \times -1}{2}\right) (-1, \frac{2 \times 4}{-1}) \right\} = \left\{ (1, -4) (3, -1) \right\}$$

$$\text{الف) } y = n^3 - 2n^2 + 2n - 1 + 1 = y = (n-1)^3 + 1 \quad (4)$$

$$y - 1 = (n-1)^3 \rightarrow \sqrt[3]{y-1} = n-1 \rightarrow n = \sqrt[3]{y-1} + 1$$

$$R_f = \mathbb{R}$$

$$\text{ب) } y = \frac{1}{n^2 - 2n} \rightarrow n^2 y - 2n y = 1 \rightarrow n^2 y - 2n y - 1 = 0$$

$$\xrightarrow{y \neq 0} n = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow 4y(y+1) \geq 0 \rightarrow \frac{-1}{+} \leq \frac{0}{+}$$

$$\xrightarrow{y=0} 0 = 1 \quad \text{ن صحتی وجود ندارد!} \rightarrow R_f = (-\infty, -1] \cup (0, +\infty)$$