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$$f(x) = \sqrt{x+4\sqrt{x-2}} \xrightarrow{\text{تبدیل با اید}} \sqrt{x-2+4\sqrt{x-2}+2} = \sqrt{(\sqrt{x-2}+2)^2} = |\sqrt{x-2}+2|$$

$$g(x) = \sqrt{x-4\sqrt{x-2}} \xrightarrow{\text{تبدیل با اید}} \sqrt{x-2-4\sqrt{x-2}+2} = \sqrt{(\sqrt{x-2}-2)^2} = |\sqrt{x-2}-2|$$

$$x > 2 \rightarrow f(x) + g(x) = a + b\sqrt{x-2} \quad f(x) + g(x) = \sqrt{x-2} + 2 + \sqrt{x-2} - 2 = 2\sqrt{x-2}$$

$$x > 2 \Rightarrow |\sqrt{x-2} + 2| = 2 \quad \text{چون} \quad 2\sqrt{x-2} = a + b\sqrt{x-2}$$

$$x > 2 \Rightarrow |\sqrt{x-2} - 2| = 0 \quad \text{چون} \quad \begin{matrix} a=0 \\ b=2 \end{matrix} \quad c=-2 \quad \frac{a+b}{c} = \frac{0+2}{-2} = -\frac{1}{2}$$

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$$f(x) = \frac{x+2}{x^2-2x+2} \rightarrow x^2-2x+2 \neq 0 \rightarrow a+b+c=0 \rightarrow \begin{matrix} x=1 \\ x=1 \end{matrix} \quad \frac{g}{f} \rightarrow f \neq 0$$

$$g(x) = \left\{ (2, 1), (1, 0), (3, 0), (0, 2) \right\} \quad f(x) = \frac{x+2}{x^2-2x+2} = \frac{x}{-1} = -x \quad f(0) = \frac{2}{2}$$

$$\text{دامنه } f = \{2, 0\}$$

$$\frac{g}{f} = \left\{ (2, \frac{1}{-2}), (0, \frac{2}{2}) \right\} \rightarrow \frac{g}{f} = \left\{ (2, -\frac{1}{2}), (0, 1) \right\}$$

3

$$f(x) = \{(1, 2), (2, -1), (3, 2), (-1, 4)\} \quad g(x) = \{(-2, 3), (1, -1), (2, 2), (-1, 0)\}$$

الف) $f(x) \rightarrow x \in D_f \rightarrow \left\{ (\frac{1}{2}, 2), (\frac{2}{3}, -1), (2, 2), (-\frac{1}{4}, 4) \right\}$

ب) $f(x^2) \rightarrow x^2 \in D_f \rightarrow \{(1, 2), (-1, 2), (\sqrt{2}, -1), (-\sqrt{2}, -1), (2, 2), (-2, 2)\} \rightarrow x^2 = -1$ ریشه حقیقی ندارد

ج) $2g^2(x) + 1 \rightarrow \{(-2, 19), (1, 3), (2, 7), (-1, 1)\}$

د) $\frac{f}{g} \rightarrow \frac{0}{0} = \{1, 3, -1\} \cdot \left\{ (1, -4), (2, -1), (-1, \frac{11}{2}) \right\} = \{(1, -4), (2, -1)\}$

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$$f(x) = \{(2, 1), (3, 1), (4, 9), (-2, 0)\} \quad h = f + g$$

$$g(x) = \{(0, 1), (2, 3), (-2, 4), (-1, 2), (3, 3)\} \quad h = \{(2, 4), (3, 4), (-2, 4)\}$$

$$\text{دامنه } h = \{2, 3, -2\}$$

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$$f(x) = \frac{\sqrt{x^2-2x}}{x-2} \quad \text{شرط } x^2-2x \geq 0 \rightarrow x(x-2) \geq 0 \rightarrow x \leq 0 \text{ یا } x \geq 2$$

$$g(x) = [x] + [-x]$$

$$D\left(\frac{f}{g+1}\right)(x) \quad \frac{f}{g+1} \rightarrow \begin{matrix} g+1 \neq 0 \\ g \neq -1 \end{matrix} \quad g(x) \rightarrow \begin{matrix} x \in \mathbb{Z} : g(x) = 0 \\ x \notin \mathbb{Z} : g(x) = -1 \end{matrix}$$

$$\text{شرط منفرد: } x-2 \neq 0 \rightarrow x \neq 2$$

$$\frac{0}{-1} + \frac{3}{1} \rightarrow [0, 3]$$

$$Df = [0, 3] - \{2\}$$

$$x \in \mathbb{Z} \cap [0, 3] - \{2\} = \{0, 1, 3\}$$

$$D\left(\frac{f}{g+1}\right)(x) = \{0, 1, 3\}$$

$f(n) = \sqrt{-n^2 + 5n - 2} \rightarrow -n^2 + 5n - 2 \geq 0 \rightarrow -n^2 + 5n - 2 = 0 \rightarrow a_1 + b + c x \rightarrow n^2/a$ $n=1, n=2 \rightarrow \frac{1}{-1} + \frac{1}{1} = 0$ $D = [1, 2]$ $g(n) = \{(1, 0), (2, 2), (3, 4), (4, 1)\}$ $h(n) = \frac{f(n)}{g(n)} \quad h(n) = \{(1, 0), (2, 0), (3, \frac{\sqrt{2}}{2}), (4, \frac{1}{2})\}$ $f(1) = 0 \quad f(2) = \sqrt{-2+10-2} = 2$ $f(3) = 0 \quad f(4) = \sqrt{-16+20-2} = 2$	6
الف) $y = n^3 - 3n^2 + 3n - 1 + 1 \rightarrow y = (n-1)^3 + 1 \quad y-1 = (n-1)^3$ $n-1 = \sqrt[3]{y-1} \rightarrow n = \sqrt[3]{y-1} + 1 \quad R = \mathbb{R}$ ب) $y = \frac{1}{n^2 - 2n} \rightarrow y n^2 - 2y n = 1$ $y n^2 - 2y n - 1 = 0 \rightarrow n = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} = \frac{1 \pm \sqrt{1+y}}{y}$ $n = \frac{1 \pm \sqrt{1+y}}{y}$ $R = (-\infty, -1] \cup (0, +\infty)$	7
الف) $y = n^2 - 4n + 10 \quad x_5 = \frac{-b}{2a} = \frac{4}{2} = 2 \quad y_5 = 4 - 8 + 10 = 6 \quad R = [6, +\infty)$ ب) $y = -n^2 + 4n + 9 \quad x_5 = \frac{-b}{2a} = \frac{-4}{-2} = 2 \quad y_5 = -9 + 18 + 9 = 18 \quad R = (-\infty, 18]$ ج) $y = \sqrt{n^2 - 4n - 5} \quad x_5 = \frac{-b}{2a} = \frac{4}{2} = 2 \quad y_5 = 4 - 8 - 5 = -9 \rightarrow [-9, +\infty) \rightarrow \sqrt{[-9, +\infty)} = [0, +\infty) = R$ د) $y = \sqrt{-n^2 + 4n} \quad x_5 = \frac{-b}{2a} = \frac{-4}{-2} = 2 \quad y_5 = -9 + 18 = 9 \rightarrow (-\infty, 9] \rightarrow R = [0, 9]$	8
الف) $y = n^3 - 8n^2 + 2n + 1 \quad R = \mathbb{R}$ ب) $y = n^3 - 4n^2 + 3n + 2 \quad R = \mathbb{R}$ ج) $y = \sqrt{n^3 - 2n^2 + 2n + 1} \rightarrow R = \sqrt{\mathbb{R}} \Rightarrow R = [0, +\infty)$ د) $y = (n^3 - 4n^2 + 3n + 1)^2 \rightarrow R = (\mathbb{R})^2 \rightarrow R = [0, +\infty)$	9
الف) $y = \frac{n+1}{n-2} \quad R = \mathbb{R} - \{2\} \quad \left\{ \frac{n+1}{n-2} = y \Rightarrow n = \frac{y-1}{y-2} \right. \quad n = \frac{y+1}{y-2} \rightarrow y \neq 2$ ب) $y = \frac{n+1}{n+1} \quad R = \mathbb{R} - \{2\} \quad \left\{ \frac{n+1}{n+1} = y \Rightarrow n = -\frac{y-1}{y-2} \right. \quad n = \frac{1-y}{y-2} \rightarrow y \neq 2$	10