

۲. اثبات حد

Subject:

نوع اول حد از حد در حد نوع اول حد

$$t = \sqrt{x} - \varepsilon \quad x + \varepsilon \sqrt{x} - \varepsilon = t^2 + \varepsilon + \varepsilon t = (t + \varepsilon)^2$$

$$x = t^2 + \varepsilon \quad f(x) = \sqrt{t^2 + \varepsilon} = t + \varepsilon$$

چون $x > 1$ داریم $(t + \varepsilon)^2 > 1$ پس $t + \varepsilon > 1$

$$g(x) \rightarrow x - \varepsilon \sqrt{x} - \varepsilon = t^2 - \varepsilon - \varepsilon t = (t - \varepsilon)^2$$

$$g(x) = t - \varepsilon$$

$$f(x) + g(x) = (t + \varepsilon) + (t - \varepsilon) = 2t$$

$$f(x) + g(x) = 2\sqrt{x} - \varepsilon \quad \leftarrow \text{چون } t = \sqrt{x} - \varepsilon$$

$$a = 0 \quad b = 2 \quad c = -\varepsilon$$

$$\frac{a+b}{c} = \frac{0+2}{-\varepsilon} = -\frac{2}{\varepsilon}$$

$$D_f = \{0, 1, 2, 3\}$$

$$f(x) \rightarrow x^2 - \varepsilon x + \varepsilon = (x-1)(x-3)$$

$$\text{بقیه } x \rightarrow x = 0, 2$$

$$x \neq 1 \quad x \neq 3$$

$$x = 0 \rightarrow g(0) = \varepsilon \quad f(0) = \frac{\varepsilon}{\sqrt{0}} \quad \frac{g(0)}{f(0)} = \frac{\varepsilon}{\frac{\varepsilon}{\sqrt{0}}} = 4$$

$$(0, 4)$$

$$x = 2 \rightarrow g(2) = 1 \quad f(2) = \frac{\varepsilon}{-1} = -\varepsilon$$

$$\frac{g(2)}{f(2)} = \frac{1}{-\varepsilon} = -\frac{1}{\varepsilon}$$

$$\frac{g}{f} = \left\{ (0, 4), \left(2, -\frac{1}{\varepsilon} \right) \right\}$$

$$الف) f(x) \quad x \rightarrow \frac{1}{x}, \frac{x}{x}, x, -\frac{1}{x}$$

$$f(x) = \left\{ \left(\frac{1}{x}, x \right), \left(\frac{x}{x}, -1 \right), (x, x), \left(-\frac{1}{x}, x \right) \right\}$$

$$ب) f(x^2) \quad x \rightarrow \pm 1, \pm \sqrt{x}, \pm x$$

$$f(x^2) = \left\{ (-x, x), (x, x), (1, x), (-1, x), (\sqrt{x}, -1), (-\sqrt{x}, -1) \right\}$$

$$ج) xg^2(x) + 1 \Rightarrow \left\{ (-x, 9), (x, 9), (1, 1), (-1, 1) \right\}$$

$$x = -x \rightarrow x(x)^2 + 1 = 9$$

$$x = 1 \rightarrow x(-1)^2 + 1 = 9$$

$$x = x \rightarrow x(x)^2 + 1 = 9$$

$$x = -1 \rightarrow x(0)^2 + 1 = 1$$

$$د) \frac{xf}{g}$$

$$xf = \left\{ (x, -x), (x, x), (-1, x), (1, x) \right\}$$

$$\frac{xf}{g} = \left\{ (-1, -x), (1, -x), (x, -1) \right\} \rightarrow \frac{xf}{g} = \left\{ (x, -1), (1, -x) \right\}$$

$$h = xf + g$$

$$x = x \rightarrow h(x) = x(1) + x = 2$$

$$x = x \rightarrow h(x) = x(1) + x = 2$$

$$x = -x \rightarrow h(-x) = x(0) + x = x$$

$$h = \left\{ (x, 2), (x, 2), (-x, x) \right\}$$

Subject:

المتغير x $x^2 - 1 = 0$
 $x = 1, 0$

المتغير y $y^2 - 1 = 0$

$y = 1, 0$

$x(y-1) = 0$

$\frac{0}{0} = \frac{0}{0}$

مخرج $x - 1 \neq 0$
 $x \neq 1$

$g(x) = [x] + [-x]$

$x \in \mathbb{Z}$
 $g(x) = 0$

$g(x) + 1 = 1$

$g(x) = -1 \leftarrow x \notin \mathbb{Z}$

مخرج $g(x) + 1$ مخرج $g(x)$

$D = \{0, 1, 2\}$

$x^2 - x - 1 = 0 \rightarrow (x-1)(x-1) = 0$
 $x^2 + 2x + 1 = 0$
 $1 \leq x \leq 2$

$R_g = \{1, 2, 3, 4\}$

$x=1 \rightarrow h(1) = \frac{\sqrt{1-1-1}}{2} = 0$

المتغير x $\{1, 2, 3\}$

$x=2 \rightarrow h(2) = \frac{\sqrt{1-2-1}}{2} = \frac{\sqrt{2}}{2}$

$x=3 \rightarrow h(3) = \frac{\sqrt{1-3-1}}{2} = 0$

$u = \{(1,0), (2, \frac{\sqrt{2}}{2}), (3,0)\}$

الف) $y = x^3 - 3x^2 + 3x \rightarrow y = (x-1)^3 + 1$ $R_g = \{1, 2\}$

ب) $y = \frac{1}{x^2 - x} \rightarrow x^2 - x = (x-1)^2 - 1 \rightarrow y = \frac{1}{(x-1)^2 - 1}$

المخرج $g(x^2 - x) = 1$

$y x^2 - x y - 1 = 0$

$\Delta > 0 \rightarrow (-xy)^2 - 4(y)(-1) > 0$

$xy^2 + 4y > 0 \rightarrow y(y+4) > 0 \rightarrow y > 0$

$R_g = (-\infty, -1] \cup (0, +\infty)$

Subject:

$$الف) y = x^2 - 4x + 10$$

ا) 0

$$\begin{cases} x = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y = 2 - 4 + 10 = 4 \end{cases}$$

$$R_y = [4, +\infty)$$

$$ب) y = -x^2 + 4x + 10$$

ا) 0

$$R_y = (-\infty, 14]$$

$$\begin{cases} x = \frac{-b}{2a} = \frac{-4}{-2} = 2 \\ y = -4 + 10 + 10 = 14 \end{cases}$$

$$ج) y = \sqrt{x^2 - 4x - 10}$$

ا) 0

$$R = \sqrt{[-1, +\infty)} = [0, +\infty)$$

$$\begin{cases} x = \frac{-b}{2a} = \frac{4}{2} = 2 \\ y = 2 - 4 - 10 = -12 \end{cases}$$

$$د) y = \sqrt{4x - x^2}$$

ا) 0

$$R = \sqrt{(-\infty, 4]} = [0, 2]$$

$$\begin{cases} x = \frac{-b}{2a} = \frac{2}{-1} = -2 \\ y = 1 - 4 = -3 \end{cases}$$

$$هـ) y = x^4 - 4x^2 + 4x + 1 \quad R_y = \mathbb{R}$$

$$و) y = x^4 - 4x^2 + 4x + 1 \quad R_y = \mathbb{R}$$

$$ز) y = \sqrt{x^4 - 4x^2 + 4x + 1} \quad R_y = \sqrt{\mathbb{R}} = [0, +\infty)$$

$$ح) y = (x^4 - 4x^2 + 4x + 1) \rightarrow R_y = [0, +\infty)$$

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و) $y = \frac{4x+1}{x-4}$ $\xrightarrow{\text{مقامیاتی}}$ $R_y = \mathbb{R} - \left\{ \frac{a}{c} \right\} \rightarrow R_y = \mathbb{R} - \{4\}$

$$y = \frac{ax+b}{cx+d} \quad ad \neq bc$$

د) $y = \frac{4x+1}{x+1}$ $\xrightarrow{\text{مقامیاتی}}$ $R_y = \mathbb{R} - \left\{ \frac{a}{c} \right\} \rightarrow R_y = \mathbb{R} - \{1\}$