

$$f(x) = \sqrt{x + k\sqrt{x - k}}$$

حل المسألة

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$$g(x) = \sqrt{x - k\sqrt{x - k}}$$

$$\frac{a+b}{c} \sqrt{-\frac{1}{F}} \quad \text{جواب}$$

$$x \geq 1 \Rightarrow f(x) + g(x) = a + b\sqrt{x + c}$$

$$t = \sqrt{x - k}$$

$$t^2 = x - k$$

$$x = t^2 + k$$

$$x + k \geq 0 \quad x \geq -k$$

$$t = \sqrt{x - k} \geq \sqrt{k} = r \rightarrow f(x) = \sqrt{(t+r)^2}$$

$$f(x) = t + r$$

$$g(x) = \sqrt{x - k\sqrt{x - k}}$$

$$g(x) = \sqrt{t^2 + k - k\sqrt{x - k}} = \sqrt{(t-r)^2} \rightarrow t \geq r \rightarrow g(x) = t - r$$

$$f(x) + g(x) = t + r + t - r = 2t \rightarrow 2\sqrt{x - k} \quad \begin{cases} a = 0 \\ b = 2 \\ c = -k \end{cases}$$

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$$f(x) = \frac{x + r}{x^2 - kx + k} \cdot \frac{k}{(x+k)(x-1)}$$

$$g(x) = \left\{ (r, 1), (1, r), (r, \omega), (r, F) \right\}$$

$$\frac{g}{f} = ?$$

$$x = r \quad g(r) = 1 \rightarrow f(r) = \frac{k}{k - r + k} = \frac{k}{2k - r}$$

$$\frac{g(r)}{f(r)} = -\frac{1}{k}$$

$$x = 1 \rightarrow g(1) = 0 \rightarrow f(1) = \frac{k}{1 - k + k} = \frac{k}{1} \Rightarrow \frac{g(1)}{f(1)} = 0$$

$$x = \omega \rightarrow g(\omega) = \omega \rightarrow f(\omega) = \frac{\omega}{\omega^2 - k\omega + k} \Rightarrow \frac{g(\omega)}{f(\omega)} = 1$$

$$x = 0 \rightarrow f(0) = \frac{0 + r}{0 - 0 + k} = \frac{r}{k} \quad \frac{g(0)}{f(0)} = \frac{0}{\frac{r}{k}} = 0 \quad (0, r) \checkmark$$

$$\frac{g}{f} = \left\{ (0, r), (r, -\frac{1}{k}) \right\}$$

$$f(x) = \begin{cases} 1 & x=1 \\ 2 & x=2 \\ 3 & x=3 \\ 4 & x=4 \end{cases}$$

$$x \in \left\{ \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2} \right\} / x \in \{1, 2, 3, 4\} \quad .3$$

$$a) f(x) = f\left(\frac{1}{2}\right) = \left\{ \left(\frac{1}{2}, 2\right), \left(\frac{3}{2}, -1\right), (1, 2), \left(-\frac{1}{2}, 4\right) \right\}$$

$$b) f(x) = f(2) = \left\{ (-2, 2), (1, 2), (-5, -1), (5, -1), (2, 2), (-1, 2) \right\}$$

$$x^2 \in D_f = \left\{ 1, 3, 5, -1 \right\}$$

$$x = \pm 5, \pm 2, \pm 1$$

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$$c) g(x) = \left\{ (-2, 4), (1, 3), (3, 4), (-1, 1) \right\}$$

$$d) \frac{f}{g} = \left\{ -1, 1, 3 \right\} \rightarrow x = -1 \rightarrow f(-1) = 4 \rightarrow g(-1) = 0$$

$$x = 1 \rightarrow f(1) = 2 \quad g(1) = -1 \quad \frac{f(1)}{g(1)} = \frac{2}{-1} = -2$$

$$x = 3 \rightarrow f(3) = -1 \quad g(3) = 2 \quad \frac{f(3)}{g(3)} = -\frac{1}{2}$$

$$\frac{f}{g} = \left\{ (1, -2), (3, -\frac{1}{2}) \right\}$$

$$h = f + g$$

$$D_f = \left\{ 1, 3, 5, -1 \right\}$$

$$D_f \cap D_g = \left\{ 1, 3, -1 \right\}$$

$$D_g = \left\{ -2, 1, -3, -1, 3 \right\}$$

$$x = 2 \rightarrow h(2) = f(2) + 2 = 4$$

$$x = 3 \rightarrow f(3) = -1, \quad g(3) = 2$$

$$h(3) = f(3) + 2 = 1$$

$$x = -2 \rightarrow f(-2) = 0, \quad g(-2) = 4$$

$$h(-2) = f(-2) + 4 = 4$$

$$h = \left\{ (2, 4), (3, 1), (-2, 4) \right\}$$

$$f(x) = \frac{1}{x-2} \quad x \neq 2$$

$$f(x) = \frac{\sqrt{1-x} - x}{x-1}$$

$$g(x) = [x] + [-x]$$

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$$D\left(\frac{f}{g+1}\right)(x) = ? \quad D_f = [0, 1] - \{1\}$$

$$1-x \geq 0 \rightarrow x \leq 1$$

$$\sqrt{1-x} - x \geq 0 \rightarrow \sqrt{1-x} \geq x \rightarrow 1-x \geq x^2 \rightarrow 1-x-x^2 \geq 0$$

$$x(1-x) \geq 0$$

$$g(x) = [x] + [-x]$$

$$[x] = x$$

$$[-x] = -x$$

$$\text{if } x \in \mathbb{Z} \rightarrow g(x) = 0$$

$$\text{if } x \notin \mathbb{Z} \rightarrow g(x) = -1$$

$$g(x) \neq -1 \rightarrow x \notin \mathbb{Z}$$

$$D = (0, 1) \quad \leftarrow \text{نقطه } x=1 \text{ را حذف می‌کنیم}$$

$$x \in [0, 1] - \{1\}$$

$$f(x) = \sqrt{1-x-x^2}$$

$$g(x) = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5)\}$$

$$h(x) = \frac{f(x)}{g(x)} = ?$$

$$x=1 \rightarrow f(1) = 0 \rightarrow g(1) = 1 \rightarrow h(1) = 0 \quad (1, 0)$$

$$x=2 \rightarrow f(2) = \sqrt{1-2-4} = \sqrt{-5} \rightarrow g(2) = 2 \rightarrow h(2) = \frac{\sqrt{-5}}{2} \quad (2, \frac{\sqrt{-5}}{2})$$

$$x=3 \rightarrow f(3) = \sqrt{1-3-9} = \sqrt{-11} = 0 \rightarrow g(3) = 3 \rightarrow h(3) = \frac{0}{3} = 0 \quad (3, 0)$$

$$x=4 \rightarrow f(4) = \sqrt{1-4-16} = \sqrt{-19} \rightarrow g(4) = 1 \quad X$$

$$h = \{(1, 0), (2, \frac{\sqrt{-5}}{2}), (3, 0)\}$$

الف $x^3 - 3x^2 + 3x \rightarrow R$

ب $y = \frac{1}{x^2 - 2x} \rightarrow y(x^2 - 2x) = 1$
 $yx^2 - 2xy = 1 \rightarrow yx^2 - 2xy - 1 = 0$
 $a = y, b = -2y, c = -1$

$\Delta = b^2 - 4ac \rightarrow (-2y)^2 - 4(y)(-1)$
 $\hookrightarrow 4y^2 + 4y = 4y(y+1)$

$y > 0 \cup y < -1$
 $y = -1/y = 0$
 $\frac{d}{dy} = -\frac{1}{y^2}$
 لا نتحيزه معقول

$y > 0 \cup y < -1$
 $R_f = (-\infty, -1) \cup (0, +\infty)$

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الف $y = x^2 - 4x + 1$ ext $\begin{matrix} 2 \\ 4 \end{matrix} \boxed{R = [0, +\infty)}$
 $a > 0$

ب $y = -x^2 + 9x + 1$ ext $\begin{matrix} 18 \\ 12 \end{matrix} \boxed{R = (-\infty, 12]}$
 $a < 0$

ج $y = \sqrt{x^2 - 4x - 3}$ ext $\begin{matrix} 2 \\ -4 \end{matrix} \rightarrow a > 0 \rightarrow \boxed{R = [0, +\infty)}$

د $y = \sqrt{4x - x^2}$ ext $\begin{matrix} 4 \\ 0 \end{matrix} \boxed{R = [0, 2]}$

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الف $R = R$
 ب $R = R$
 ج $R = [0, +\infty)$
 د $R = [0, +\infty)$

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الف $\frac{x^3 + 1}{x - 2} \rightarrow a = 3, c = 1 \rightarrow R = \{3\}$

ب $y = \frac{x + 1}{x + 1} \rightarrow a = 2, c = 1 \rightarrow R = \{2\}$