

نام و نام خانوادگی: شماره: ۷ کلاس: (معمول)

$$f(x) = \sqrt{x+4}\sqrt{x-4}, g(x) = \sqrt{x-4}\sqrt{x-4}, f(x)+g(x) = a+b\sqrt{x+c}, \frac{a+b}{c} = ?$$

$$x-4 = (\sqrt{x-4})^2 \Rightarrow t = \sqrt{x-4} \Rightarrow x = t^2+4 \rightarrow f(x) = \sqrt{t^2+4+4} = \sqrt{(t+2)^2} = t+2 \Rightarrow f(x) = t+2$$

$$\hookrightarrow g(x) = \sqrt{t^2+4-4} = \sqrt{t^2} = |t-2| \xrightarrow{x \geq 4} x-4 \geq 0 \Rightarrow \sqrt{x-4} \geq 0 \rightarrow t \geq 2 \rightarrow g(x) = t-2$$

$$\frac{f(x)+g(x)}{(t+2)+(t-2)} = 2t \xrightarrow{t=\sqrt{x-4}} 2\sqrt{x-4} = a+b\sqrt{x+c} \Rightarrow \begin{cases} a=0 \\ b=2 \\ c=-4 \end{cases} \Rightarrow \frac{a+b}{c} = \frac{0+2}{-4} = -\frac{1}{2}$$

$$f(x) = \frac{x+2}{x^2-4x+3}, g(x) = \{(2,1), (1,0), (3,5), (0,4)\}$$

$$f(x) = \frac{x+2}{(x-1)(x-3)} \rightarrow D_f = \mathbb{R} - \{1, 3\}$$

$$f(2) = \frac{2+2}{2^2-4\cdot 2+3} = -\frac{4}{5} \rightarrow \frac{g(2)}{f(2)} = \frac{1}{-\frac{4}{5}} = -\frac{5}{4}$$

$$f(0) = \frac{0+2}{0^2-4\cdot 0+3} = \frac{2}{3} \rightarrow \frac{g(0)}{f(0)} = \frac{4}{\frac{2}{3}} = 6$$

$$\frac{g}{f} = \{(2, -\frac{5}{4}), (0, 6)\}$$

$$f(x) = \{(1,2), (3,-1), (4,2), (-1,4)\}, g(x) = \{(-2,3), (1,-1), (3,2), (-1,0)\}$$

$$\text{الف) } f(2x) = \{(\frac{1}{2}, 2), (\frac{3}{2}, -1), (2, 2), (\frac{1}{2}, 4)\} \rightarrow 2x \in \{1, 3, 4, -1\}$$

$$\text{ب) } f(x^2) \Rightarrow x \in D_f = \{1, 3, 4, -1\} \rightarrow \{(-2, 2), (-1, 2), (1, 2), (2, 2)\} = f(x^2)$$

$$\text{ج) } 2g(2x+1) \xrightarrow{g(1)=1 \rightarrow 2(-1)+1=-1, g(3)=2 \rightarrow 2(1)+1=3, g(-1)=0 \rightarrow 2(0)+1=1} \{(-2, 1), (1, 3), (3, 9), (-1, 1)\} = 2g(2x+1)$$

$$\text{د) } \frac{f}{g} \Rightarrow \frac{f}{g} = \{(1, 4), (3, -2), (4, 4), (-1, 2)\} \rightarrow \frac{f}{g} \Rightarrow \text{دامنه مشترک: } 1, 3 \Rightarrow \frac{f}{g} = \{(1, -4), (3, -1)\}$$

$$f(x) = \{(2,1), (3,1), (4,9), (-2,0)\}, g(x) = \{(5,1), (2,3), (-2,4), (3,3), (-1,2)\}, h = 2f + g$$

$$2f = \{(2,2), (3,2), (4,18), (-2,0)\}$$

$$2f + g \rightarrow 2, 3, -2: \text{دامنه مشترک}$$

$$2f + g = \{(2, 5), (3, 5), (-2, 4)\}$$

$$f(x) = \frac{\sqrt{3x-x}}{x-2}, g(x) = [x] + [-x], D\left(\frac{f}{g+1}\right)(x) = ?$$

$$\forall x \geq 0 \rightarrow x \geq 0: \sqrt{3x-x} \geq 0 \rightarrow \sqrt{3x} \geq x \xrightarrow{x \geq 0} 3x \geq x^2 \rightarrow x^2-3x \leq 0 \rightarrow x(x-3) \leq 0 \rightarrow 0 \leq x \leq 3$$

$$x-2 \neq 0 \rightarrow x \neq 2 \Rightarrow D_f = [0, 3] - \{2\} \quad \text{I}$$

$$g(x) = \begin{cases} 0 & x \in \mathbb{Z} \\ -1 & x \notin \mathbb{Z} \end{cases} \rightarrow g(x)+1 = \begin{cases} 1 & x \in \mathbb{Z} \\ 0 & x \notin \mathbb{Z} \end{cases} \rightarrow g(x)+1 \neq 0 \rightarrow x \in \mathbb{Z} \quad \text{II}$$

$$I \cap II = \{0, 1, 3\} = D_{\frac{f}{g+1}}$$

$$f(x) = \sqrt{5x - x^2}, g(x) = \{(1,3), (2,2), (4,4), (5,1)\}, h(x) = \frac{f(x)}{g(x)}$$

$$-x^2 + 5x - 4 \geq 0 \rightarrow x^2 - 5x + 4 \leq 0 \rightarrow (x-1)(x-4) \leq 0 \rightarrow 1 \leq x \leq 4 \rightarrow D_f = [1, 4]$$

$$D_g = \{1, 2, 4, 5\} \Rightarrow D_f \cap D_g = \{1, 2, 4\}$$

$$h = \{(1,0), (2, \frac{\sqrt{2}}{2}), (4,0)\}$$

$$\begin{aligned} f(1) &= \sqrt{5-1-4} = \sqrt{0} = 0 \\ f(2) &= \sqrt{10-4-4} = \sqrt{2} \\ f(4) &= \sqrt{20-16-4} = \sqrt{0} = 0 \end{aligned}$$

الف) $y = x^3 - 3x^2 + 3x \Rightarrow (x-1)^3 + 1 = y \rightarrow x-1 = \sqrt[3]{y-1} \rightarrow x = \sqrt[3]{y-1} + 1 \} R = \mathbb{R}$
 $(x-1)^3 = x^3 - 3x^2 + 3x - 1$
 $(-\infty, +\infty)$

ب) $y = \frac{1}{x^2 + 2x} \rightarrow yx^2 - 2xy = 1 \rightarrow yx^2 - 2xy - 1 = 0 \rightarrow \Delta = 4y^2 + 4y \rightarrow \Delta \geq 0$
 $4y^2 + 4y \geq 0 \rightarrow 4y(y+1) \geq 0 \rightarrow y(y+1) \geq 0 \rightarrow y = -1 \} \frac{-1 \pm 0}{+1-1} \rightarrow (-\infty, -1], [0, +\infty)$
 $y = 0$
 $\rightarrow R = (-\infty, -1] \cup (0, +\infty)$

الف) $y = x^2 - 4x + 1 \rightarrow \min \left| \frac{-b}{2a} = \frac{4}{2} = 2 \right| R = [4, +\infty)$

ب) $y = -x^2 + 4x + 3 \rightarrow \max \left| \frac{-b}{2a} = \frac{4}{-2} = -2 \right| R = (-\infty, 12]$

ج) $y = \sqrt{x^2 - 4x - 3} \rightarrow \min \left| \frac{4}{-2} = -2 \right| R = [-7, +\infty) \xrightarrow{\sqrt{\quad}} [0, +\infty) = \mathbb{R}$

د) $y = \sqrt{x^2 + 4x + 5} \rightarrow \max \left| \frac{4}{2} = 2 \right| R = (-\infty, 9] \xrightarrow{\sqrt{\quad}} R = [0, 3]$

الف) $y = x^3 - 5x^2 + 2x + 1 \rightarrow R = (-\infty, +\infty)$

ب) $y = x^3 - 9x^2 + 3x + 2 \rightarrow R = (-\infty, +\infty)$

ج) $y = \sqrt{x^3 - 9x^2 + 5x + 1} \rightarrow R = [0, +\infty) \rightarrow$ از زیر رادیکال اعداد مثبت بیرون آیند

د) $y = (x^3 - 4x^2 + 3x + 1)^2 \rightarrow R = [0, +\infty) \rightarrow$ اعداد صحیح بین ۰ و ۲ رسیده مثبت اند.

الف) $y = \frac{3x+1}{x-2} \rightarrow R = \mathbb{R} - \left\{ \frac{a}{c} \right\} \rightarrow R = \mathbb{R} - \{3\}$

ب) $y = \frac{2x+1}{x+1} \rightarrow R = \mathbb{R} - \{2\}$