

۲. آنالیز به روش

$$\rightarrow f(x) = \sqrt{x+4\sqrt{x-4}} = \sqrt{x-4+4\sqrt{x-4}+4} = \sqrt{(\sqrt{x-4}+2)^2} = |\sqrt{x-4}+2| \leq \sqrt{x-4}+2$$

معادله مشتق است چون $x \geq 4$ است پس از مشتق کردن بدون تغییر علامت میاد

$$\rightarrow g(x) = \sqrt{x-4\sqrt{x-4}} = \sqrt{x-4-4\sqrt{x-4}+4} = \sqrt{(\sqrt{x-4}-2)^2} = |\sqrt{x-4}-2| \geq \sqrt{x-4}-2$$

$$\Rightarrow g(x) + f(x) = \sqrt{x-4}+2 + \sqrt{x-4}-2 = 2\sqrt{x-4} = a + b\sqrt{x+c} \Rightarrow \begin{cases} a=0 \\ b=2 \\ c=-4 \end{cases} \Rightarrow \frac{a+b}{c} = \frac{2}{-4} = -\frac{1}{2}$$

$$\rightarrow f(x) = \frac{x+2}{(x-1)(x-3)} \rightarrow f(1) = \frac{3}{(1)(-1)} = -3 \checkmark$$

$$f(1) = \frac{3}{0} \Rightarrow x$$

$$f(3) = \frac{5}{0} \Rightarrow x$$

$$f(0) = \frac{2}{(-3)(-1)} = \frac{2}{3} \checkmark$$

$$\Rightarrow \begin{cases} \frac{g}{f}(1) = -\frac{1}{3} \\ \frac{g}{f}(0) = 4 \end{cases} \rightarrow \left\{ \left(1, -\frac{1}{3}\right), \left(0, 4\right) \right\}$$

الف) $f(x) = \left\{ \left(\frac{1}{2}, 2\right), \left(\frac{3}{2}, -1\right), (2, 2), \left(-\frac{1}{2}, 4\right) \right\}$

باید به دقت اول و دوم را بنویسیم.

ب) $f(x^2) = ? \rightarrow \begin{cases} x^2 \leq 1 \rightarrow x \leq \pm 1 \\ x^2 \leq 3 \rightarrow x \leq \pm\sqrt{3} \\ x^2 \leq 4 \rightarrow x \leq \pm 2 \\ x^2 \leq 9 \rightarrow x \leq \pm 3 \end{cases} \Rightarrow \left\{ (1, 2), (-1, 2), (\sqrt{3}, -1), (-\sqrt{3}, -1), (2, 2), (-2, 2) \right\}$

ج) $2g^2(x) + 1 = \left\{ (-2, 19), (1, 3), (2, 9), (-1, 1) \right\}$

باید مولفه ها را بنویسیم و به توان ۲ برسانیم و بعد منجمد کنیم و بعد به اضافه یک کنیم.

د) $\frac{2f}{g} = \left\{ \left(1, \frac{4}{-1}\right), \left(2, \frac{2(-2)}{2}\right), \left(-1, \frac{8}{-3}\right), \left(2, -1\right), \left(2, -1\right) \right\}$

ه) $h(x) = 2f(x) + g(x) \rightarrow \text{مشتق } x = -2, 2, 3 \Rightarrow \left\{ (-2, 4), (2, 5), (3, 5) \right\}$

و) $\left(\frac{f}{g+1}\right)(x) = \frac{\frac{\sqrt{x^2-x}}{x-2}}{[x]+[x]+1} \rightarrow \begin{cases} \sqrt{x^2-x} \geq 0 \rightarrow \sqrt{x} \geq x \rightarrow x^2 \geq x^2 \rightarrow x^2 - x \leq 0 \rightarrow x(x-1) \leq 0 \\ \rightarrow x \in [0, 1] \end{cases}$

$x \notin \mathbb{Z} \Rightarrow [x] + [-x] = -1 \rightarrow$ چون صحیح منفرجه $\Rightarrow x \in \mathbb{Z}$

$x \in \mathbb{Z} \Rightarrow [x] + [-x] = 0 \rightarrow$ صحیح $\Rightarrow x \in \mathbb{Z}$

$\frac{3}{+1-1+1} = \frac{3}{1} = 3$

ز) $D = \left\{ 0, 1 \right\}$

$$f(x) = \sqrt{5x - x^2 - 8} = \sqrt{-(x-1)(x+8)} \Rightarrow \frac{-8-1}{-1+1} \Rightarrow x \neq -2, -2, -1, 0 \rightarrow \text{hms } \{(1,0), (8,0), (2, \sqrt{4})\}$$

$$\text{الف) } y = x^3 - 2x^2 + 3x \xrightarrow{\text{تفاضل و مساوی}} y = x^3 - 2x^2 + 3x + 1 - 1 = (x-1)^3 + 1 \rightarrow x = \sqrt[3]{y-1} + 1 \rightarrow R_f: \mathbb{R}$$

$$\text{ب) } \rightarrow yx^2 - 2xy = 1 \rightarrow yx^2 - 2xy - 1 = 0 \xrightarrow{\Delta} x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow 4y(y+1) \geq 0 \rightarrow \frac{-1 \pm \sqrt{1+4y}}{2} \rightarrow R_f: (-\infty, -1] \cup (0, +\infty)$$

$$\text{الف) } a > 0 \rightarrow \min_{x \in \mathbb{R}} \Rightarrow \text{ext} \left| \frac{-b}{2a} = 2 \right| \rightarrow [4, +\infty) \subset R_f$$

$$\text{ب) } a < 0 \rightarrow \max_{x \in \mathbb{R}} \Rightarrow \text{ext} \left| \frac{-b}{2a} = 12 \right| \rightarrow (-\infty, 12] \subset R_f$$

$$\text{ج) } a > 0 \rightarrow \min_{x \in \mathbb{R}} \Rightarrow \text{ext} \left| \frac{-b}{2a} = 2 \right| \xrightarrow{\sqrt{}} \sqrt{[-1, +\infty)} = [0, +\infty)$$

$$\text{د) } a < 0 \rightarrow \max_{x \in \mathbb{R}} \Rightarrow \text{ext} \left| \frac{-b}{2a} = 3 \right| \xrightarrow{\sqrt{}} \sqrt{(-\infty, 9]} \subset [0, 3]$$

الف) $\mathbb{R}$ ب) $\mathbb{R}$
چون برکتین توان عبارت، فرد هست.

ج) $\mathbb{R} \xrightarrow{\sqrt{}} [0, +\infty)$ د) $\mathbb{R} \xrightarrow{\sqrt{}} [0, +\infty)$
چون زوج، اشیای مثبت.

$$\text{الف) } \rightarrow \mathbb{R} - \left\{ \frac{a}{c} \right\} = \mathbb{R} - \{3\}$$

$$\text{ب) } \rightarrow \mathbb{R} - \left\{ \frac{a}{c} \right\} = \mathbb{R} - \{2\}$$