

$f(x) = \sqrt{x + \sqrt{x - x}}$      $g(x) = \sqrt{x - \sqrt{x - x}}$      $\Rightarrow 1$      $f(x) + g(x) = a + b\sqrt{x + c}$      $\frac{a+b}{c} = ?$   
 $\hookrightarrow (A+B)^2 = A^2 + B^2 + 2AB$   
 $A = \sqrt{x - x}$      $B = \sqrt{x - x}$      $\hookrightarrow (\sqrt{x - x} + \sqrt{x - x})^2 = (\sqrt{x - x})^2 + (\sqrt{x - x})^2 + 2(\sqrt{x - x} \times \sqrt{x - x}) = (x - x) + (x - x) + 2(x - x) = x + \sqrt{x - x}$   
 $f(x) = \sqrt{(\sqrt{x - x} + \sqrt{x - x})^2} \rightarrow x \geq 1 \rightarrow \sqrt{x - x} \geq 1 \rightarrow f(x) = \sqrt{x - x} + \sqrt{x - x}$   
 $\hookrightarrow (\sqrt{x - x} - \sqrt{x - x})^2 = (\sqrt{x - x})^2 + (\sqrt{x - x})^2 - 2(\sqrt{x - x} \times \sqrt{x - x}) = (x - x) + (x - x) - 2(x - x) = x - \sqrt{x - x}$   
 $g(x) = \sqrt{(\sqrt{x - x} - \sqrt{x - x})^2} \rightarrow x \geq 1 \rightarrow \sqrt{x - x} \geq 1 \rightarrow g(x) = \sqrt{x - x} - \sqrt{x - x}$   
 $f(x) + g(x) = \sqrt{x - x} + \sqrt{x - x} + \sqrt{x - x} - \sqrt{x - x} = 2\sqrt{x - x} = a + b\sqrt{x + c}$   
 $a = 0$   
 $b = 2$   
 $c = -x$   
 $\frac{a+b}{c} = \frac{0+2}{-x} = \frac{2}{-x} = -\frac{1}{x}$

$f(x) = \frac{x + 1}{x^2 - x + 1}$      $g(x) = \{(1, 1)(1, 0)(1, 5)(0, 4)\}$      $\frac{g}{f} = \{(1, -\frac{1}{x})(0, 4)\}$   
 $x = 1 \rightarrow g(1) = 1 \rightarrow f(1) = \frac{1}{1 - 1 + 1} = \frac{1}{1} = 1$      $\frac{g(1)}{f(1)} = \frac{1}{1} = 1$      $(1, 1)$   
 $x = 1 \rightarrow g(1) = 0 \rightarrow f(1) = \frac{1}{1 - 1 + 1} = \frac{1}{1} = 1$      $\frac{g(1)}{f(1)} = \frac{0}{1} = 0$      $(1, 0)$   
 $x = 1 \rightarrow g(1) = 5 \rightarrow f(1) = \frac{1}{1 - 1 + 1} = \frac{1}{1} = 1$      $\frac{g(1)}{f(1)} = \frac{5}{1} = 5$      $(1, 5)$   
 $x = 0 \rightarrow g(0) = 4 \rightarrow f(0) = \frac{0 + 1}{0^2 - 0 + 1} = \frac{1}{1} = 1$      $\frac{g(0)}{f(0)} = \frac{4}{1} = 4$      $(0, 4)$

$f(x) = \{(1, 1)(1, -1)(1, 2)(1, 4)\}$      $g(x) = \{(1, 1)(1, -1)(1, 2)(1, 4)\}$   
 $\hookrightarrow f(x) \rightarrow \begin{cases} x = 1 \rightarrow \frac{1}{1} = 1 & (1, 1) \\ x = -1 \rightarrow \frac{1}{1 - 1 + 1} = 1 & (1, -1) \\ x = 2 \rightarrow \frac{1}{1 - 2 + 1} = \frac{1}{0} \text{ undefined} & (1, 2) \\ x = -2 \rightarrow \frac{1}{1 - 4 + 1} = \frac{1}{-2} & (1, 4) \end{cases}$   
 $\hookrightarrow \frac{g}{f} \rightarrow \begin{cases} x = 1 \rightarrow \frac{1}{1} = 1 & (1, 1) \\ x = -1 \rightarrow \frac{1}{1 - 1 + 1} = 1 & (1, -1) \\ x = 2 \rightarrow \frac{1}{1 - 2 + 1} = \frac{1}{0} \text{ undefined} & (1, 2) \\ x = -2 \rightarrow \frac{1}{1 - 4 + 1} = \frac{1}{-2} & (1, 4) \end{cases}$   
 $\hookrightarrow \frac{g}{f} \rightarrow \begin{cases} x = 1 \rightarrow \frac{1}{1} = 1 & (1, 1) \\ x = -1 \rightarrow \frac{1}{1 - 1 + 1} = 1 & (1, -1) \\ x = 2 \rightarrow \frac{1}{1 - 2 + 1} = \frac{1}{0} \text{ undefined} & (1, 2) \\ x = -2 \rightarrow \frac{1}{1 - 4 + 1} = \frac{1}{-2} & (1, 4) \end{cases}$

$f(x) = \{(1, 1)(1, 2)(1, 4)(1, 8)\}$      $g = \{(1, 1)(1, 2)(1, 4)(1, 8)\}$      $h = f \cdot g \Rightarrow \{(1, 1)(1, 2)(1, 4)(1, 8)\}$   
 $\rightarrow x = 1 \rightarrow \begin{cases} f(1) = 1 & x = 1 \\ g(1) = 1 & x = 1 \\ h(1) = 1 + 1 = 2 & x = 1 \end{cases}$      $(1, 2)$   
 $\rightarrow x = 2 \rightarrow \begin{cases} f(2) = 2 & x = 2 \\ g(2) = 2 & x = 2 \\ h(2) = 2 + 2 = 4 & x = 2 \end{cases}$      $(2, 4)$   
 $\rightarrow x = 4 \rightarrow \begin{cases} f(4) = 4 & x = 4 \\ g(4) = 4 & x = 4 \\ h(4) = 4 + 4 = 8 & x = 4 \end{cases}$      $(4, 8)$   
 $\rightarrow x = 8 \rightarrow \begin{cases} f(8) = 8 & x = 8 \\ g(8) = 8 & x = 8 \\ h(8) = 8 + 8 = 16 & x = 8 \end{cases}$      $(8, 16)$

$f(x) = \frac{\sqrt{x^2 - x}}{x - 1}$      $g(x) = [x] + [-x]$      $(\frac{f}{g+1})(x)$      $\frac{f}{g+1}$   
 $\hookrightarrow x \geq 0 \rightarrow x \geq 0$   
 $\sqrt{x^2 - x} \geq 0 \rightarrow (\sqrt{x^2 - x})^2 \geq 0 \rightarrow x^2 - x \geq 0 \rightarrow x(x - 1) \geq 0 \rightarrow x \geq 1$   
 $x - 1 \neq 0 \rightarrow x \neq 1$   
 $D_f = [0, 1] \cup [1, \infty)$   
 $D_g = [0, 1] \cup [1, \infty)$   
 $D(\frac{f}{g+1}) = [0, 1] \cup [1, \infty)$

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| $P_m = \sqrt{ax - x^2 - t}$<br>$\lim_{x \rightarrow \infty} (ax - x^2 - t) > 0 \rightarrow x^2 - ax + t \leq 0 \rightarrow (x-1)(x-t) = 0$<br>$\downarrow x=1 \quad \downarrow x=t$<br>$n=1 \checkmark$<br>$n=r \checkmark$<br>$n=t \checkmark$<br>$x=a \checkmark$                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | $g(x) = \{(1,0)(r,r)(t,t)(a,1)\}$<br>$h(x) = \frac{P(x)}{g(x)} \rightarrow \{(1,0)(r,\frac{r}{r})(t,0)\}$<br>$D_f = 1 \leq x \leq t$<br>$h(x) \rightarrow \begin{cases} x=1 \rightarrow P(1) = \sqrt{a-1-t} = \sqrt{0} = 0 & h(1) = \frac{0}{1} = 0 \quad (1,0) \\ x=r \rightarrow P(r) = \sqrt{10-t-t} = \sqrt{r} & h(r) = \frac{\sqrt{r}}{r} = \frac{1}{\sqrt{r}} \quad (r, \frac{1}{\sqrt{r}}) \\ x=t \rightarrow P(t) = \sqrt{10-1-t} = \sqrt{0} = 0 & h(t) = \frac{0}{t} = 0 \quad (t,0) \end{cases}$ | 6  |
| $\text{a)} y = x^r - r x^r + r^n \rightarrow (x-1)^r + 1 = y \rightarrow y-1 = (x-1)^r \rightarrow x-1 = \sqrt[r]{y-1} \rightarrow x = \sqrt[r]{y-1} + 1$<br>$R = \mathbb{R}$<br>$\text{b)} y = \frac{1}{x^r - r x} \rightarrow x^r - r x = \frac{1}{y} \rightarrow x^r - r x - \frac{1}{y} = 0$<br>$\alpha = \frac{-b \pm \sqrt{\Delta}}{2a}$<br>$\Delta = b^2 - 4ac \rightarrow (-r)^2 - 4(-\frac{1}{y}) = r^2 + \frac{4}{y}$<br>$\Delta > 0 \rightarrow r^2 + \frac{4}{y} > 0 \rightarrow 1 + \frac{1}{y} > 0$<br>$\rightarrow \frac{1}{y} > -1 \rightarrow \frac{1}{y} + 1 > 0 \rightarrow \frac{1+y}{y} > 0$<br>$\rightarrow +1 + \frac{1}{y} > 0 \rightarrow \frac{1}{y} > -1 \rightarrow +\frac{1}{y} > -1$<br>$\frac{-1}{+1} < \frac{0}{-\frac{1}{y}} < +$ | $\alpha = \frac{-b \pm \sqrt{\Delta}}{2a}$<br>$\Delta = b^2 - 4ac \rightarrow (-r)^2 - 4(-\frac{1}{y}) = r^2 + \frac{4}{y}$<br>$\Delta > 0 \rightarrow r^2 + \frac{4}{y} > 0 \rightarrow 1 + \frac{1}{y} > 0$<br>$\rightarrow \frac{1}{y} > -1 \rightarrow \frac{1}{y} + 1 > 0 \rightarrow \frac{1+y}{y} > 0$<br>$\rightarrow +1 + \frac{1}{y} > 0 \rightarrow \frac{1}{y} > -1 \rightarrow +\frac{1}{y} > -1$<br>$\frac{-1}{+1} < \frac{0}{-\frac{1}{y}} < +$                                             | 7  |
| $\text{a)} y = x^r - r x + 1$<br>$\alpha = \frac{-b}{2a} = \frac{r}{r} = 1$<br>$y = r^r - r + 1 = 9 \quad (1,4) \quad \alpha = 1 > 0$<br>$\text{b)} y = -x^r + 4x + 1$<br>$\alpha = \frac{-b}{2a} = \frac{-4}{-r} = 1$<br>$y = -(1)^r + 4 + 1 = 12 \quad (1,12) \quad \alpha = -1 < 0$<br>$\text{c)} } \sqrt{x^r - r x - r} > 0$<br>$\alpha = \frac{-b}{2a} = \frac{r}{r} = 1$<br>$y = r - 1 - r = -1$<br>$R = [0, +\infty)$<br>$\text{d)} } \sqrt{4x - x^r} > 0$<br>$\alpha = \frac{-b}{2a} = \frac{-4}{-r} = 1$<br>$y = 1 - 1 - 1 = -1$<br>$\sqrt{4} = 2$<br>$R = [0, 1^r]$                                                                                                                                                                                      | $\text{a)} } \sqrt{x^r - r x - r} > 0$<br>$\alpha = \frac{-b}{2a} = \frac{r}{r} = 1$<br>$y = r - 1 - r = -1$<br>$R = [0, +\infty)$<br>$\text{b)} } \sqrt{4x - x^r} > 0$<br>$\alpha = \frac{-b}{2a} = \frac{-4}{-r} = 1$<br>$y = 1 - 1 - 1 = -1$<br>$\sqrt{4} = 2$<br>$R = [0, 1^r]$                                                                                                                                                                                                                        | 8  |
| $\text{a)} } y = \frac{x^r - ax^r + r x + 1}{x^r}$<br>$R = \mathbb{R}$<br>$\text{b)} } y = \frac{x^r - 4x^r + r x + 1}{x^r}$<br>$R = \mathbb{R}$<br>$\text{c)} } y = \sqrt{\frac{x^r - 4x^r + r x + 1}{x^r}}$<br>$R = [0, +\infty)$<br>$\text{d)} } y = (\frac{x^r - r x^r + r x + 1}{x^r})^r > 0$<br>$R = [0, +\infty)$                                                                                                                                                                                                                                                                                                                                                                                                                                           | $\text{a)} } y = \frac{x^r - ax^r + r x + 1}{x^r}$<br>$R = \mathbb{R}$<br>$\text{b)} } y = \frac{x^r - 4x^r + r x + 1}{x^r}$<br>$R = \mathbb{R}$<br>$\text{c)} } y = \sqrt{\frac{x^r - 4x^r + r x + 1}{x^r}}$<br>$R = [0, +\infty)$<br>$\text{d)} } y = (\frac{x^r - r x^r + r x + 1}{x^r})^r > 0$<br>$R = [0, +\infty)$                                                                                                                                                                                   | 9  |
| $\text{a)} } y = \frac{r x + 1}{x - r}$<br>$\alpha = r$<br>$C = 1$<br>$R = \mathbb{R} - \{\frac{a}{C}\} = \mathbb{R} - \{\frac{r}{1}\} = \mathbb{R} - \{r\}$<br>$\text{b)} } y = \frac{r x + 1}{x + 1}$<br>$\alpha = r$<br>$C = 1$<br>$R = \mathbb{R} - \{\frac{a}{C}\} = \mathbb{R} - \{\frac{r}{1}\} = \mathbb{R} - \{r\}$                                                                                                                                                                                                                                                                                                                                                                                                                                       | $\text{a)} } y = \frac{r x + 1}{x - r}$<br>$\alpha = r$<br>$C = 1$<br>$R = \mathbb{R} - \{\frac{a}{C}\} = \mathbb{R} - \{\frac{r}{1}\} = \mathbb{R} - \{r\}$<br>$\text{b)} } y = \frac{r x + 1}{x + 1}$<br>$\alpha = r$<br>$C = 1$<br>$R = \mathbb{R} - \{\frac{a}{C}\} = \mathbb{R} - \{\frac{r}{1}\} = \mathbb{R} - \{r\}$                                                                                                                                                                               | 10 |