

۱۹۷۵ آزمون نهایی

$$\begin{aligned} g(x) &= \sqrt{x-4} \sqrt{x-4} \xrightarrow{\text{ساده کنیم}} \sqrt{(\sqrt{x-4}-2)^2} \rightarrow \sqrt{x-4}-2 \\ f(x) &= \sqrt{x+4} \sqrt{x-4} \xrightarrow{\text{ساده کنیم}} \sqrt{(\sqrt{x-4}+2)^2} \rightarrow \sqrt{x-4}+2 \end{aligned}$$

$$\begin{aligned} \star f(x)+g(x) &= a+b\sqrt{x+c} \\ \hookrightarrow A-2+A+2 &= 2A \rightarrow 2\sqrt{x-4} \end{aligned} \quad \left. \begin{aligned} a=0 \quad b=2 \quad c=-4 \\ \star \frac{a+b}{c} &= \frac{2}{-4} = -\frac{1}{2} \end{aligned} \right\} \sim 2$$

$$f(x) = \frac{x+2}{x^2-x+3} \xrightarrow{\text{تجزیه کنیم}} \frac{x+2}{(x-1)(x-3)} \xrightarrow{\text{تجزیه کنیم}} \frac{1}{x-1} - \frac{2}{x-3} \quad (-\infty, 1) \cup (3, +\infty) : \text{دامنه}$$

$$\begin{aligned} g(x) &= \left\{ \left(\frac{1}{2}, 1 \right), (1, 0), (3, 0), (0, 6) \right\} \\ \hookrightarrow f(2) &= \frac{4}{1 \cdot (-1)} = -4 \quad f(0) = \frac{2}{(-1) \cdot (-3)} = \frac{2}{3} \end{aligned} \quad \left. \begin{aligned} \frac{g}{f} &= \left\{ \left(2, -\frac{1}{4} \right), (0, 6) \right\} \\ \text{در این جا} & \end{aligned} \right\} \sim 2$$

$$2x=1 \rightarrow x=\frac{1}{2} \quad 2x=3 \rightarrow x=\frac{3}{2} \quad 2x=4 \rightarrow x=2 \quad 2x=-1 \rightarrow x=-\frac{1}{2}$$

$$\text{الف) } f(2x) = \left\{ \left(\frac{1}{2}, 2 \right), \left(\frac{3}{2}, -1 \right), (2, 2), \left(-\frac{1}{2}, -4 \right) \right\}$$

$$\text{ب) } f(x^2) = \left\{ (1, 2), (-1, 2), (\sqrt{3}, -1), (-\sqrt{3}, -1), (2, 2), (-2, 2) \right\} \quad x^2 = \frac{1}{4} \rightarrow x = \pm \frac{1}{2} \quad x^2 = 3 \rightarrow x = \pm \sqrt{3} \quad x^2 = 4 \rightarrow x = \pm 2$$

$$\text{ج) } 2g^2(x)+1 = \left\{ (-2, 19), (1, 3), (3, 9), (-1, 1) \right\} \quad 2x^2+1=19 \rightarrow x^2=9 \rightarrow x=\pm 3 \quad 2x^2+1=3 \rightarrow x^2=1 \rightarrow x=\pm 1$$

$$\text{د) } \frac{2f}{g} = \left\{ (1, -4), (3, -1), \left(-\frac{1}{2}, -8 \right) \right\} \rightarrow \left\{ (1, -4), (3, -1) \right\} \quad \frac{2x^2}{-1} = -4 \rightarrow x^2 = 2 \rightarrow x = \pm \sqrt{2} \quad \frac{2x^2}{-1} = -1 \rightarrow x^2 = \frac{1}{2} \rightarrow x = \pm \frac{1}{\sqrt{2}}$$

$$2x+1+3=6 \quad 2x+1+3=6 \quad 2x+6=6 \quad \text{محورهای مشترک: } -2, 3, 2$$

$$h = 2f+g = \left\{ (2, 5), (3, 5), (-2, -4) \right\} \sim 2$$

$$g(x) = [x] + [-x] \quad \begin{cases} x \notin \mathbb{Z} \rightarrow g(x) = -1 \\ x \in \mathbb{Z} \rightarrow g(x) = 0 \end{cases} \Rightarrow \text{درمجموعه } h \text{ تابع } g+1 \rightarrow g(x) = -1 \rightarrow -1+1=0 \quad \text{ن.ن. } x$$

$$f(x) = \frac{\sqrt{3x-x^2}}{x-2} \xrightarrow{\text{تجزیه کنیم}} \begin{cases} 1. \sqrt{3x-x^2} > 0 \rightarrow x > 0 \quad [0, +\infty) \\ 2. \sqrt{3x-x^2} > 0 \rightarrow (\sqrt{3x-x^2})^2 > 0 \rightarrow 3x-x^2 > 0 \rightarrow 0 < x < 3 \\ 3. x-2 \neq 0 \rightarrow x \neq 2 \quad [0, 3] \end{cases} \quad \text{دامنه } [0, 3] - \{2\} \in D_f$$

$$\star D\left[\frac{f}{g+1}\right] = \left\{ \frac{0}{1} = 0, \frac{1}{1} = 1, \frac{3}{1} = 3 \right\} \sim 2$$

$$f(x) = \sqrt{5x - x^2 - 4} \geq 0 \rightarrow 5x - x^2 - 4 \geq 0 \rightarrow (1-x)(x-4) \geq 0 \rightarrow [1, 4] = D_f$$

$$h(x) = \frac{f(x)}{g(x)} = \left\{ (1,0), (2, \frac{\sqrt{2}}{2}), (4,0) \right\} \text{ ن.ج.}$$

$$f(x) = \sqrt{1-x-x^2} = \sqrt{2}$$

$$\frac{f(x)}{g(x)} = \frac{\sqrt{2}}{2}$$

ملاحظات: 1 و 2 و 3

$$5x-1-4=0 / 10-x-x^2=2 / 20-16-x=0$$

1, \sqrt{2}

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الف) $y = x^3 - 3x^2 + 3x - 1 + 1 \rightarrow (x-1)^3 + 1 \rightarrow y-1 = (x-1)^3 \rightarrow \sqrt[3]{y-1} + 1 = x \rightarrow \mathbb{R} = R_f$
 (مربع 1+2) b) c) توان 3 تا نثری ندارد.

ب) $y = \frac{1}{x^2 - 2x} \rightarrow y(x^2 - 2x) - 1 = 0 \rightarrow x = \frac{2y \pm \sqrt{4y^2 + 4y}}{2y} \rightarrow y(y+1) \neq 0$
 $\Delta = 4y^2 + 4y$
 $a = b^2 - 4ac$
 $y(y) \neq 0$

$$\frac{-1 \pm \sqrt{1+4y}}{2y}$$

$(-\infty, -1] \cup (0, +\infty)$
 $y \neq 0$

$$R_f = (-\infty, -1] \cup (0, +\infty) \text{ ن.ج.}$$

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الف) $y = x^2 - 4x + 10 \rightarrow \text{ext}_{\min} : \left| \frac{-b}{2a} = \frac{4}{2} = 2 \right|$
 $a > 0 \text{ min}$
 $y = 6$
 $R_f = [6, +\infty) \text{ ن.ج.}$

ب) $y = -x^2 + 4x + 3 \rightarrow \text{ext}_{\max} : \left| \frac{-b}{2a} = \frac{-4}{-2} = 2 \right|$
 $a < 0 \text{ max}$
 $y = 11$
 $R_f = (-\infty, 11] \text{ ن.ج.}$

ج) $y = \sqrt{x^2 - 4x - 3} \rightarrow \text{ext}_{\min} : \left| \frac{-b}{2a} = \frac{4}{2} = 2 \right|$
 $a > 0 \text{ min}$
 $y = -1$
 $R_f = [0, +\infty) \text{ ن.ج.}$

د) $y = \sqrt{4x - x^2} \rightarrow \text{ext}_{\max} : \left| \frac{-b}{2a} = \frac{-4}{-2} = 2 \right|$
 $a < 0 \text{ max}$
 $y = 2$
 $R_f = [0, 2] \text{ ن.ج.}$

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الف) $R_f = \mathbb{R}$

ب) $R_f = \mathbb{R}$

ج) $R_f = [0, +\infty)$

د) $R_f = [0, +\infty)$

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الف) $y = \frac{x+1}{x-2} \rightarrow R_f = \mathbb{R} - \{2\} \text{ ن.ج.}$

ب) $y = \frac{x+1}{x+1} \rightarrow R_f = \mathbb{R} - \{2\}$

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$$Df \rightarrow \omega_n - u^r - \varepsilon \gamma. \rightarrow u^r - \omega_n + \varepsilon \gamma. \rightarrow 1 \leq n \leq 3$$

(9)

$$Df \cap Dg = \{1, 2, 3\}$$

$$\frac{f}{g} = h = \left\{ (1, 0) \left(2, \sqrt{\frac{2}{3}} \right) \left(3, 0 \right) \right\}$$