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آزمون بهاره

$$f(x) = \sqrt{x+4}\sqrt{x-4} - 4 + 4 \rightarrow \sqrt{(x-4)+4+4}\sqrt{x-4} = \sqrt{(\sqrt{x-4}+2)^2} = |\sqrt{x-4}+2|$$

$$g(x) = \sqrt{x-4}\sqrt{x-4} \rightarrow \sqrt{(x-4)-2}\sqrt{x-4} = |\sqrt{x-4}-2| \rightarrow g(x) = \begin{cases} \sqrt{x-4}+2 & \text{اگر } \sqrt{x-4}+2 \geq 0 \\ \sqrt{x-4}-2 & \text{اگر } \sqrt{x-4}-2 < 0 \end{cases}$$

$$f(x) + g(x) = a + b\sqrt{x+c}$$

$$\begin{cases} a=0 \\ b=2 \\ c=-4 \end{cases} \quad \frac{a+b}{c} = \frac{1}{-4} \quad \text{پ}$$

$$g(x) = \{(2,1), (1,0), (3,5), (0,4)\} \text{ و } f(x) = \frac{x+2}{x^2-4x+3}$$

$$\frac{g}{f} = \{(0,4), (2, -\frac{1}{4})\} \quad \text{پ}$$

$$\begin{aligned} f(0) &\rightarrow \frac{2}{3} & g(0) &= 4 & f(2) &= -4 \\ g(0) &= 4 & f(2) &= -4 & g(2) &= 1 \end{aligned}$$

$$f(x) = \frac{1}{x} \quad x=1, 3, 5, -1 \quad f(2) = \left\{ \left(\frac{1}{4}, 2\right), \left(\frac{4}{9}, -1\right), (2, 2), \left(-\frac{1}{4}, 4\right) \right\}$$

$$f(x^2) \quad x^2=1 \rightarrow x=\pm 1 \quad x^2=4 \rightarrow x=\pm 2 \quad x^2=9 \rightarrow x=\pm 3 \quad f(x^2) = \left\{ (1,2), (-1,2), (2,2), (-2,2), (3,2), (-3,2), (1,-1), (-1,-1), (2,-1), (-2,-1), (3,-1), (-3,-1) \right\}$$

$$g(x) = x^2 + 1 \quad g(2) = 5, g(3) = 10, g(-1) = 2, g(0) = 1 \quad \left\{ (-2, 19), (1, 3), (3, 9), (-1, 2) \right\}$$

$$\frac{f}{g} = \left\{ (-1, -\frac{1}{2}), (1, \frac{1}{2}), (3, -\frac{1}{3}), (-3, \frac{1}{3}) \right\}$$

$$\{2, 3, -1\}$$

$$x=2 \rightarrow h(2) = 2f(2) + g(2) = 2+3=5$$

$$x=3 \rightarrow h(3) = 2(1) + 3=5$$

$$x=-1 \rightarrow h(-1) = 2(0) + 2=2$$

$$h = \{(2,5), (3,5), (-1,2)\} \quad \text{پ}$$

$$f(x) = \frac{\sqrt{3x-x}}{x-2} \rightarrow 3x \geq 0 \rightarrow x \geq 0$$

$$\sqrt{3x-x} \geq 0 \rightarrow \sqrt{2x} \geq 0 \rightarrow 2x \geq 0 \rightarrow x \geq 0$$

$$D_f = [0, 2) \cup (2, \infty)$$

$$g(x) = [x] + [-x]$$

$$[x] + [-x] = 0$$

$$x \notin \mathbb{Z} \rightarrow [x] + [-x] = -1$$

$$D_{\frac{f}{g}} = \{0, 2\}$$

$$\frac{f}{g} = \{0, 2\}$$

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$2x - x^2 - 4 \geq 0 \rightarrow x^2 - 2x + 4 \leq 0$ $\Delta = 4 - 16 = -12 < 0$ $f(x) = x^2 - 2x + 4$ $f(1) = 1 - 2 + 4 = 3 > 0$ $f(2) = 4 - 4 + 4 = 4 > 0$ $f(4) = 16 - 8 + 4 = 12 > 0$ $\{ (0, 4) \cup (\frac{\sqrt{12}}{2}, 2) \cup (4, \infty) \}$	6
الف) $x^3 - 3x^2 + 4x - 1 = 0$ $(x-1)^3 + 1 \rightarrow (x-1)^3 = y-1 \rightarrow x = \sqrt[3]{y-1} + 1$ $R_f = \mathbb{R}$ ب) $y x^2 - 2xy = 1$ $y x^2 - 2xy - 1 = 0 \rightarrow \Delta \geq 0 \rightarrow 4y^2 + 4y \geq 0 \rightarrow 4y(y+1) \geq 0$ $y \geq 0 \rightarrow y \in [0, \infty)$ $y < -1 \rightarrow y \in (-\infty, -1]$	7
الف) $y = x^2 - 4x + 10$ $-\frac{b}{2a} = \frac{4}{2} = 2 \rightarrow x^2 - 8x + 10 = 0$ $[4, \infty)$ ب) $y = -x^2 + 6x + 3$ $-\frac{b}{2a} = \frac{-6}{-2} = 3 \rightarrow -x^2 + 12x + 3 = 0$ $(-\infty, 12]$ ج) $y = \sqrt{x^2 - 4x - 3}$ $-\frac{b}{2a} = \frac{4}{2} = 2 \rightarrow x^2 - 8x - 3 = 0 \rightarrow R_f = [0, \infty)$ د) $y = \sqrt{6x - x^2}$ $-\frac{b}{2a} = \frac{-6}{-2} = 3 \rightarrow 12 - 9 = 3 \rightarrow \sqrt{9} = 3$ $[0, 3]$	8
الف) $\mathbb{R}$ ب) $\mathbb{R}$ ج) $[0, \infty)$ د) $[0, \infty)$	9
الف) $\frac{a}{c} = 2 \rightarrow \mathbb{R} - \{2\}$ ب) $\frac{a}{c} = 2 \rightarrow \mathbb{R} - \{2\}$	10

$$f(n) = \frac{\sqrt{\sqrt{r}n} - n}{n - r} \rightarrow \begin{cases} n \neq r \\ n \geq 0 \end{cases} \quad (\sqrt{r}n) \rightarrow D_f = [0, r] - \{r\} \quad (\infty) \\ \sqrt{r}n - n \geq 0 \rightarrow r \geq n^2 \rightarrow 0 \leq n \leq r$$

$$g(n) + 1 \neq 0 \rightarrow g(n) \neq -1 \rightarrow [n] + [-n] \neq -1 \rightarrow n \in \mathbb{Z}$$

$$D_{\frac{f}{g+1}} = \left\{ D_f \cap D_g, g(n) \neq -1 \right\} = \left\{ 0, 1, r \right\} \\ \downarrow \\ [0, r] - \{r\} \cap \mathbb{Z}$$