یازدهم دفتر A — تمارین — تکلیف شماره ۱

۱) $A \times B = \{(1,2),(1,3),(2,2),(2,3),(3,2),(3,3)\}$

$B^2 = \{(2,2),(2,3),(3,2),(3,3)\}$

$A \times B - B^2 = \{(1,2),(1,3)\}$

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۲) الف) $m^2 = m + 2 \rightarrow m^2 - m - 2 = 0$

$(m-2)(m+1) = 0 \rightarrow \begin{cases} m = -1 \\ m = 2 \end{cases}$

چون اگر $m=2$ باشد ، $(2,4)$ میشه که چون $(2,1)$ $\leftarrow m \neq 2$

را داریم به ازای ۲ باید یک عدد داشته باشیم. ⇐

$\boxed{m = -1}$

ب) $m^2 = m + 2 \rightarrow m^2 - m - 2 = 0 \rightarrow \begin{cases} m = -1 \\ m = 2 \end{cases}$

$m \neq 2$ طبق ق ، $m \neq -1$ طبق ق

⇐ پس به ازای هیچ مقدار m این مجموعه تابع نمی باشد.

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۳) $\boxed{a \times b = 8 \times -8 = -64}$

$x = 1 \rightarrow y = x^2 - 1 = 1 - 1 = 0$

$x = 2 \rightarrow y = x^3 = 8$

$ax + b \xrightarrow{x=1} a + b = 0$, $\xrightarrow{x=2} 2a + b = 8$ $\Big\}$ $a = 8 \Rightarrow b = -8$

(۴)

$x = k \rightarrow \quad 4 - 3k = 2k - 3$

$7 = 5k \rightarrow k = \frac{7}{5}$

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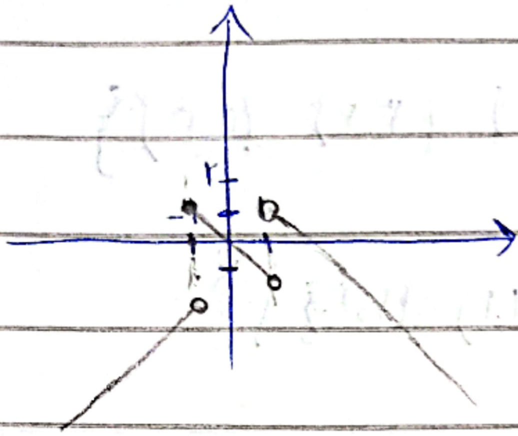
(۵)



$D_f = \mathbb{R} - \{1\}$

$R_f = (-\infty, 1]$

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(۶)

الف) $2y^3 + x^2 + 1 = 0$

$$\left.\begin{array}{l} 2y_1^3 = -x^2 - 1 \\ 2y_2^3 = -x^2 - 1 \end{array}\right\} \Rightarrow 2y_1^3 = 2y_2^3$$

$y_1^3 = y_2^3 \Rightarrow y_1 = y_2$

تابع هست

ب) $x^2 + 4y^2 - 2x - 12y + 10 = 0$

$\hookrightarrow x^2 + 4y^2 - 2x - 12y + 1 + 9 = 0$

$(x^2 - 2x + 1) + (4y^2 - 12y + 9) = 0$

$(x-1)^2 + (y - \frac{3}{2})^2 = 0$

$\hookrightarrow x - 1 = 0 \Rightarrow x = 1 \quad , \quad y - \frac{3}{2} = 0 \Rightarrow y = \frac{3}{2}$

نقطه $(1, \frac{3}{2})$ در فضای ما

[illegible] تابع هست.

(7)

الف) $x\sin y = y \times \sin x$

$\rightarrow x=\pi \Rightarrow \sin x = 0$

$\pi \times \sin y = y \times 0 \rightarrow y = k\pi$ پس تابع نیست.

$\llcorner y=\pi \, , y = 2\pi , \dots$

ب) $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2$

$\left(\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}}\right)^2 = 4 \rightarrow \frac{x}{y} + \frac{y}{x} + 2 \times \underbrace{\sqrt{\frac{x}{y}} \times \sqrt{\frac{y}{x}}}_{1} = 4$

$\frac{x}{y} + \frac{y}{x} = 2 \rightarrow \frac{x^2+y^2}{xy} = 2 \rightarrow x^2+y^2-2xy=0$

$(x-y)^2 = 0 \Rightarrow x=y$ تابع هست.

(8)

الف) $y = \sqrt{\frac{x-1}{x-3}} + \sqrt{\frac{2-x}{x}} \rightarrow$ $\underset{0}{|}\;+\;\underset{2}{|}$ $D_f = (0,2]$ (1)

$\underset{1}{|} \;-\; \underset{3}{|}$ ($+$, $-$, $+$) $D_f = (-\infty, 1] \cup (3, +\infty)$ (2)

$(1) \cap (2) \Rightarrow D_f = (0, 1]$

ب) $\sqrt{x} + \sqrt{y-1} = \sqrt{3} \rightarrow \sqrt{y-1} = \sqrt{3} - \sqrt{x}$

$x \geq 0$ (1)

$\sqrt{3} - \sqrt{x} \geq 0$

$\sqrt{3} \geq \sqrt{x}$

$3 \geq x$ (2)

$(1) \cap (2) \Rightarrow D_f = [0, 3]$

(ب۱۱) $\frac{\sqrt{x^2-7x+4}}{\sqrt{x^2-10x+14}}$ $\begin{cases} x=4 \\ x=1 \end{cases}$

$(-\infty,1]\cup[4,+\infty)$ ①

$\sqrt{x^2-10x+14}$ $\begin{cases} \\ \end{cases}$

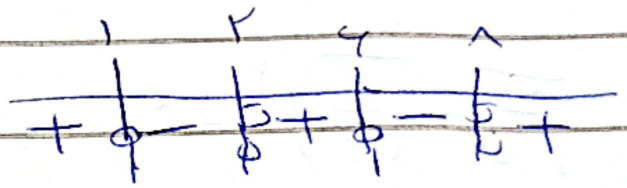
$-9x+14\geq 0$

$14\geq 9x$

$\frac{14}{9}\geq x$ ②

① ∩ ② $\Rightarrow D_f=(-\infty,1]$

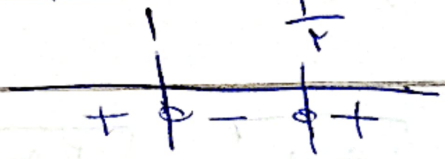
(ب) $\sqrt{\frac{x^2-7x+4}{x^2-10x+14}}$ $\begin{cases} x=4 \\ x=1 \end{cases}$



$\begin{cases} x=2 \\ x=8 \end{cases}$

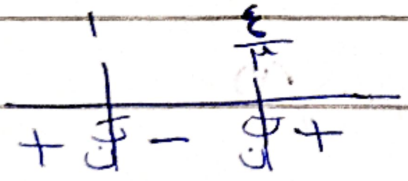
$D_f=(-\infty,1]\cup(2,4]\cup(8,+\infty)$

(ب۱۲) $y=\frac{\sqrt{2x^2-5x+2}}{\sqrt{4x^2-7x+3}}$ $\begin{cases} x=1 \\ x=\frac{2}{2} \end{cases}$



$(-\infty,1]\cup[\frac{2}{2},+\infty)$ ①

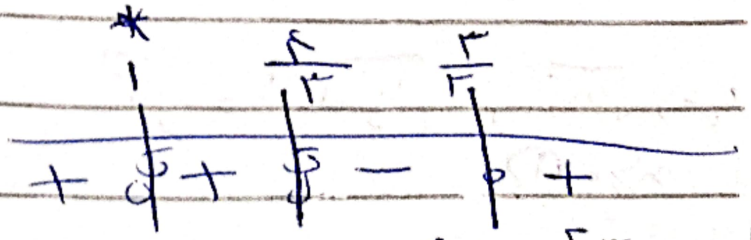
$\begin{cases} x=1 \\ x=\frac{2}{4} \end{cases}$



$(-\infty,1)\cup(\frac{2}{4},+\infty)$ ②

① ∩ ② $\Rightarrow D_f=(-\infty,1)\cup[\frac{2}{4},+\infty)$

(ب) $\sqrt{\frac{2x^2-5x+2}{2x^2-7x+3}}$



$D_f=(-\infty,1)\cup(1,\frac{3}{4})\cup[\frac{2}{2},+\infty)$