

$y_1 = y_2 \rightarrow x_1 = x_2 \rightarrow m = 3$
 $x^2 - x - 2 = 0 \rightarrow \begin{matrix} 2 \\ -1 \end{matrix} \rightarrow \begin{matrix} 2 \\ -1 \end{matrix}$
 $f = \{(3, 2), (2, 5), (3, 2), (3, 2), (-1, 4)\}$ (نکته)

$m = 2$
 $m - 1 = 2 - 1 = 1 \rightarrow a = -1$
 $a + 2 = -1 + 2 = 1 \rightarrow n = 2$ جواب

$x^2 - 1 \rightarrow \frac{x+1}{x-1} \rightarrow R = [2, +\infty)$
 $x + a \rightarrow \frac{x+1}{x-1} \rightarrow R = (-\infty, 1 + a)$ (نکته: برای $a < 1$ به سمت راست) $\rightarrow 1 + a < 2 \rightarrow a < 1 \rightarrow (-\infty, 1)$

$x^2 - 1 \rightarrow \frac{x+1}{x-1} \rightarrow R = [2, +\infty)$
 $a x + a - 1 \rightarrow \frac{x+1}{x-1} \rightarrow R = (-\infty, a - 1] \rightarrow a - 1 < 2 \rightarrow a < 3$
 $a > 0 \Rightarrow a > 0$
 $\left. \begin{matrix} a < 3 \\ a > 0 \end{matrix} \right\} \rightarrow 0 < a < 3 \rightarrow (0, 3)$

یک به یک است (نکته)
 $y_1 = y_2 \rightarrow x_1 = x_2 \rightarrow x^3 + 2 = x^2 + 2 \rightarrow x_1 = x_2 \rightarrow$
 $x^3 = y - 2 \rightarrow x = \sqrt[3]{y - 2} \rightarrow y = \sqrt[3]{x - 2} \rightarrow f^{-1} = \sqrt[3]{x - 2}$
 (ب) در صورتی که میانه ضابطه، متعلق به یک می باشد است بنابراین نمی توانیم به یک به یک معکوس هم بنهارد

افق تابع همگرافیک یک به یک است
 $y x - 2 y = x + 1 \rightarrow x(y - 2) = 2y + 1 \rightarrow x = \frac{2y + 1}{y - 2} \Rightarrow y = \frac{x + 1}{x - 2}$
 $\rightarrow f^{-1} = \frac{x + 1}{x - 2}$
 (ب) تابع همگرافیک یک به یک است
 $y x + 2 y = 4 + 2 x \rightarrow x(y - 2) = 4 - 2 y \rightarrow x = \frac{4 - 2 y}{y - 2}$
 $\rightarrow y = \frac{4 - 2 x}{x - 2} \rightarrow f^{-1} = \frac{4 - 2 x}{x - 2}$

$$y_1 = y_2 \rightarrow \sqrt{x_1 - 2} = \sqrt{x_2 - 2} \xrightarrow{\text{بمربع}} x_1 = x_2 \quad \text{تساوی است}$$

$$y^2 = x - 2 \rightarrow x = y^2 + 2 \rightarrow y = x^2 + 2 \rightarrow f^{-1} = x^2 + 2$$

در حالت کلی میسر است ولی چون دایره دارای سه نقطه است

$$y = x^2 - 4x + 2 + 1 \rightarrow y = (x - 2)^2 - 1 \rightarrow (x - 2)^2 = y + 1 \rightarrow x - 2 = \pm \sqrt{y + 1}$$

$$\rightarrow x = \pm \sqrt{y + 1} + 2 \rightarrow y = \sqrt{x + 1} + 2 \rightarrow f^{-1} = \sqrt{x + 1} + 2$$

$$y = x + \sqrt{x} \rightarrow y = (\sqrt{x})^2 + \sqrt{x} \rightarrow \Delta$$

$$\sqrt{x} = \frac{-1 \pm \sqrt{1 + 4y}}{2} \rightarrow x = \left(\frac{-1 \pm \sqrt{1 + 4y}}{2} \right)^2$$

$$\rightarrow x = \frac{1 + 1 + 4y - 2\sqrt{1 + 4y}}{4} \rightarrow x = \frac{4y + 2 - 2\sqrt{1 + 4y}}{4} \rightarrow x = y + \frac{1}{2} - \frac{1}{2}\sqrt{1 + 4y} \Rightarrow f(x)^{-1} = x + \frac{1}{2} - \frac{1}{2}\sqrt{1 + 4x}$$

$$y = ax + b - \sqrt{x} - c = x + \frac{1}{2} - \frac{1}{2}\sqrt{1 + 4x} \rightarrow \begin{cases} a = 1 \\ b = \frac{1}{2} \\ c = \frac{1}{2} \end{cases} \rightarrow 1 + 1 + 1 = (3)$$

$$\frac{x_1}{\sqrt{x_1^2 - 2}} = \frac{x_2}{\sqrt{x_2^2 - 2}} \xrightarrow{\text{بمربع}} \frac{x_1^2}{x_1^2 - 2} = \frac{x_2^2}{x_2^2 - 2} \rightarrow x_1^2 x_2^2 - 2x_1^2 = x_1^2 x_2^2 - 2x_2^2 \rightarrow x_1 = x_2 \quad \text{تساوی است}$$

$$y = \frac{x}{\sqrt{x^2 - 2}} \rightarrow x = \frac{y}{\sqrt{y^2 - 2}} \rightarrow x^2 = \frac{y^2}{y^2 - 2} \rightarrow x^2 y^2 - 2x^2 = y^2 \rightarrow x^2 y^2 - y^2 = 2x^2$$

$$\rightarrow y^2 (x^2 - 1) = 2x^2 \rightarrow y^2 = \frac{2x^2}{x^2 - 1} \rightarrow y = \pm \sqrt{\frac{2x^2}{x^2 - 1}} \rightarrow y = \frac{\sqrt{2}x}{\sqrt{x^2 - 1}}$$

$$f^{-1} = \frac{x}{1+x} \quad \left\{ \begin{array}{l} x \geq 0 \rightarrow f^{-1} = \frac{x}{1+x} \xrightarrow{\text{معکوس}} x = \frac{y}{y+1} \rightarrow yx + x = y \rightarrow y(x-1) = x \rightarrow y = \frac{x}{x-1} \\ x < 0 \rightarrow f^{-1} = \frac{x}{1-x} \xrightarrow{\text{معکوس}} x = \frac{y}{1-y} \rightarrow x - yx = y \rightarrow y(x+1) = x \rightarrow y = \frac{x}{x+1} \end{array} \right.$$

$$f\left(-\frac{1}{2}\right) = \frac{-\frac{1}{2}}{\frac{1}{2} + 1} = \frac{-\frac{1}{2}}{\frac{3}{2}} = \left(-\frac{1}{3}\right)$$

$$g(x) = \sqrt{x} - 1 \rightarrow \sqrt{x} = y + 1 \xrightarrow{\text{بمربع}} x = y^2 + 1 + 2y \rightarrow x = y^2 + 1 + 2y \rightarrow g^{-1}\left(-\frac{1}{2}\right) = \frac{1}{4} + 1 - \frac{1}{2} = \frac{3}{4}$$

$$\frac{10x - \frac{1}{x} + \frac{9x^2}{x^2 + 10}}{\frac{1}{x^2 + 10}} = \frac{-20 + \frac{1}{x^2 + 10}}{\frac{1}{x^2 + 10}} = \left(-\frac{20}{x^2 + 10}\right) \quad \text{جواب نهایی}$$

$$y = \sqrt[3]{x-1} \rightarrow y^3 = x-1 \rightarrow x = y^3 + 1 \rightarrow y = x^3 + 1 \rightarrow f^{-1} = x^3 + 1$$

$$g(x) = x^2 + 1 + \sqrt{x^2 + 1} \rightarrow g^{-1}(x) \rightarrow g(x) = 12 \rightarrow x^2 + 1 + \sqrt{x^2 + 1} = 12 \rightarrow (x^2 + 1) + \sqrt{x^2 + 1} = 11$$

$$\rightarrow \sqrt{x^2 + 1} = \frac{-1 \pm \sqrt{1 + 41}}{2} \rightarrow \frac{-1 \pm \sqrt{42}}{2} \xrightarrow{\text{بمربع}} \rightarrow \sqrt{x^2 + 1} = 3 \rightarrow x^2 + 1 = 9 \rightarrow x^2 = 8$$

$$\rightarrow x = \sqrt{8} \rightarrow g^{-1}(12) = \sqrt{8}$$