

الف) $f = \{(3, 2), (n, 6), (2, n^2 - n), (m, 2), (-1, 4)\}$

$2 = n^2 - n \Rightarrow n^2 - n - 2 = 0 \Rightarrow n^2 - 2n + n - 2 = 0 \Rightarrow n(n-2) + 1(n-2) = 0 \Rightarrow (n-2)(n+1) = 0$
 $n = 2$ یا $n = -1$

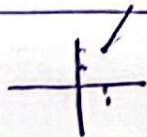
ب) $f = \{(-1, 1), (1, 2), (2, 3), (a, m-1), (a+2, n), (m, 3)\}$

$m \geq 2$

$m-1 \geq 1 \Rightarrow a \geq -1$

$a+2 \geq 1 \Rightarrow n \geq 2$

$f(n) = \begin{cases} 2n-1 & ; n \geq 1 \\ n+a & ; n < 1 \end{cases}$



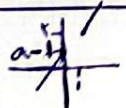
$n \in \mathbb{Z}, m-1, R = [2, +\infty)$

$n+a < 2 \Rightarrow 1+a < 2 \Rightarrow a < 1 \Rightarrow$

$a \in (-\infty, 1)$

$n \geq 1 \rightarrow 2n-1 \geq 1 \Rightarrow (2n-1) \cap (n+a) = \emptyset$

$f(n) = \begin{cases} 2n-1 & ; n \geq 1 \\ a+n+a-1 & ; n < 0 \end{cases}$



$n \geq 1, y \in [2, +\infty)$
 $0 < n, y \in (-\infty, a-1]$

$n=1 \Rightarrow 2-1=1$

$n=0 \Rightarrow a-1$

$a \neq 0 \Rightarrow$ شیب صاف است و شیب ۱ نیست

$\left. \begin{matrix} n \geq \emptyset < a \\ a-1 < 2 \Rightarrow a < 3 \end{matrix} \right\} n \geq 0 < a < 3$

الف) $y = n^2 + 2 \Rightarrow n_1^2 + 2 = n_2^2 + 2 \Rightarrow n_1^2 = n_2^2 \Rightarrow n_1 = n_2 \rightarrow$ یک به یک است

معکوس: $n = y^2 + 2 \Rightarrow y^2 = n - 2 \Rightarrow y = \sqrt{n-2} \Rightarrow f^{-1}(n) = \sqrt{n-2}$

ب) $y = n^2 - 4n + 2 \Rightarrow$ نمودار سهمی $\Rightarrow$ یک به یک نیست

الف) $y = \frac{2n+1}{n-2} \Rightarrow$ یک به یک $\rightarrow$ همگرا نیست

معکوس: $n = \frac{2y+1}{y-2} \Rightarrow y(n-2) = 2y+1 \Rightarrow -1-2n = 2y - yn \Rightarrow -1-2n = y(2-n) \Rightarrow$

$y = \frac{-1-2n}{2-n} \Rightarrow y = \frac{2n+1}{n-2} \Rightarrow f^{-1}(n) = \frac{2n+1}{n-2}$

ب) $y = \frac{2+2n}{n+2} \Rightarrow$ تابع ثابت ۲ $\rightarrow$ یک به یک نیست

الف) $y = \sqrt{n-2} \Rightarrow$ یک به یک $\rightarrow$

معکوس: $n = y^2 + 2 \Rightarrow n^2 = y^2 + 2 \Rightarrow y = \sqrt{n-2} \Rightarrow f^{-1}(n) = \sqrt{n-2}$

ب) $n^2 - 4n + 3, n \in [2, +\infty) \Rightarrow$ راس سهمی $\frac{-b}{2a} = \frac{4}{2} = 2 \Rightarrow$ یک به یک است

$n = y^2 - 4n + 3 + 1 - 1 \Rightarrow n = (y-2)^2 - 1 \Rightarrow n-1 = (y-2)^2 \Rightarrow \sqrt{n-1} = y-2$

$y = \sqrt{n-1} + 2 \Rightarrow f^{-1}(n) = \sqrt{n-1} + 2$

$f(n) = n + \sqrt{n}$, $f^{-1}(n) = an + b - \sqrt{n} - c$, $\sqrt{n} = t \Rightarrow f(n) = t^2 + t$ ①
 $a + 2b + c = ?$ $f^{-1}(n) \Rightarrow y = t^2 + t + \frac{1}{t} - \frac{1}{t} \Rightarrow (t + \frac{1}{t})^2 - \frac{1}{t^2} = y \Rightarrow y = (\sqrt{n} + \frac{1}{\sqrt{n}})^2 - \frac{1}{n}$
 $D_f = [0, +\infty) = R_{f^{-1}} \Rightarrow n = (\sqrt{y} + \frac{1}{\sqrt{y}})^2 - \frac{1}{y} \Rightarrow n + \frac{1}{y} = (\sqrt{y} + \frac{1}{\sqrt{y}})^2 \Rightarrow$
 $\pm \sqrt{n + \frac{1}{y}} = \sqrt{y} + \frac{1}{\sqrt{y}} \Rightarrow \sqrt{n + \frac{1}{y}} - \frac{1}{\sqrt{y}} = \sqrt{y} \Rightarrow y = n + \frac{1}{y} - \sqrt{n + \frac{1}{y}} \Rightarrow \begin{cases} a = 1 \\ b = \frac{1}{2} \\ c = -\frac{1}{2} \end{cases}$
 $a + 2b + c = 1 + 1 - 1 = 1$

$y = \frac{n}{\sqrt{n^2 - 2}}$, $\frac{n_1}{\sqrt{n_1^2 - 2}} = \frac{n_2}{\sqrt{n_2^2 - 2}} \xrightarrow{*2} \frac{n_1^2}{n_1^2 - 2} = \frac{n_2^2}{n_2^2 - 2} \Rightarrow n_1^2 \sqrt{n_2^2 - 2} = n_2^2 \sqrt{n_1^2 - 2}$ ②
 $\sqrt{n_1^2 - 2} = \sqrt{n_2^2 - 2} \Rightarrow n_1^2 = n_2^2 \Rightarrow n_1 = \pm n_2$
 لیکن نمی توان در n مختلف علامت داشته باشد یعنی $n_1 = n_2$
 معکوس: $f^{-1}(n) = \frac{y}{\sqrt{y^2 - 2}} \xrightarrow{*2} n^2 = \frac{y^2}{y^2 - 2} \Rightarrow n^2 y^2 - 2n^2 = y^2 \Rightarrow n^2 y^2 - y^2 = 2n^2$
 $y^2(n^2 - 1) = 2n^2 \Rightarrow y^2 = \frac{2n^2}{n^2 - 1} \Rightarrow y = \pm \sqrt{\frac{2n^2}{n^2 - 1}} = \frac{y \cdot n}{\sqrt{n^2 - 1}}$

$f^{-1}(n) = \frac{n}{1 + |n|}$, $g(n) = \sqrt{n} - 1$, $g^{-1}(-\frac{r}{\omega}) + f(-\frac{r}{\omega})$ ③
 $g^{-1}(n) \Rightarrow \sqrt{n} - 1 \Rightarrow y + 1 = \sqrt{n} \Rightarrow n = (y + 1)^2 \Rightarrow g^{-1}(n) = (n + 1)^2 \Rightarrow g^{-1}(-\frac{r}{\omega}) = (-\frac{r}{\omega} + 1)^2 = \frac{r}{r\omega}$
 $f^{-1}(n) = \begin{cases} \frac{n}{1+n} ; n \geq 0 \\ \frac{n}{1-n} ; n < 0 \end{cases} \Rightarrow y = \frac{n}{1-n} \Rightarrow f(n) \Rightarrow n = \frac{y}{1-y} \Rightarrow n - ny = y \Rightarrow n = y + ny$
 $n = y(1 + n) \Rightarrow y = \frac{n}{1+n} \Rightarrow f(-\frac{r}{\omega}) = \frac{-\frac{r}{\omega}}{1 + \frac{r}{\omega}} = \frac{-\frac{r}{\omega}}{\frac{\omega + r}{\omega}} = \frac{-r}{\omega + r}$
 $g^{-1}(-\frac{r}{\omega}) + f(-\frac{r}{\omega}) = \frac{r}{r\omega} - \frac{r}{\omega + r} = \frac{-r^2}{r\omega(\omega + r)}$

$g(n) = f(n) + \sqrt{f(n)}$, $f^{-1}(n) = \sqrt{n-1}$ ④
 $f^{-1}(n) = y \Rightarrow y^2 = n-1 \Rightarrow y^2 - 1 = n \Rightarrow f(n) = n^3 - 1$
 $g(n) = n^3 - 1 + \sqrt{n^3 - 1} \Rightarrow t = \sqrt{n^3 - 1} \Rightarrow g(n) = t^2 + t \Rightarrow y = t^2 + t \Rightarrow t^2 + t - y = 0$
 $t = \frac{-1 \pm \sqrt{1 + 4y}}{2} \xrightarrow{\sqrt{1 + 4y} \geq 0} t = \frac{-1 + \sqrt{1 + 4y}}{2}$
 $n^3 + 1 = t^2 \Rightarrow n^3 = t^2 - 1 \Rightarrow n = \sqrt[3]{t^2 - 1} \Rightarrow n = \sqrt[3]{\left(\frac{-1 + \sqrt{1 + 4y}}{2}\right)^2 - 1} \Rightarrow g^{-1}(n) = \sqrt[3]{\left(\frac{-1 + \sqrt{1 + 4y}}{2}\right)^2 - 1}$
 $g^{-1}(11) = \sqrt[3]{\left(\frac{-1 + \sqrt{1 + 44}}{2}\right)^2 - 1} = \sqrt[3]{8} = 2$