

الف) $\{(3, 2), (n, 5), (3, n^2 - n), (m, 2), (-1, 4)\}$

$n^2 - n = 2 \rightarrow n^2 - n - 2 = 0 \rightarrow (n - 3)(n + 1) = 0 \rightarrow \begin{cases} n = 3 \\ n = -1 \end{cases} \rightarrow \text{حذف}$

$\xrightarrow{n=3} \{(3, 2), (2, 5), (m, 2), (-1, 4)\} \rightarrow m = 3$

ب) $F = \{(-1, 1), (1, 2), (2, 3), (a, m-1), (a+2, n), (m, 3)\} \quad m = 2$

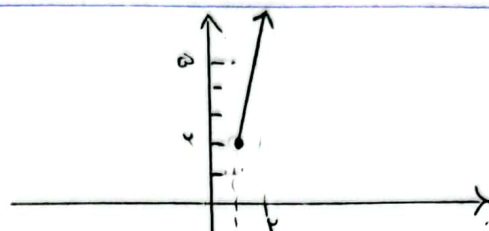
$F = \{(-1, 1), (1, 2), (2, 3), (a, 1), (a+2, n)\} \rightarrow a = -1, n = 2$

$f(x) = \begin{cases} 3x - 1 & x \geq 1 \\ x + a & x < 1 \end{cases}$

برای $x < 1$ $\xrightarrow{+a} x + a < 1 + a$
برابر: $x + a < 2$

$1 + a = 2$
 $a = 1$

$\rightarrow \text{محدوده: } (-\infty, 1)$

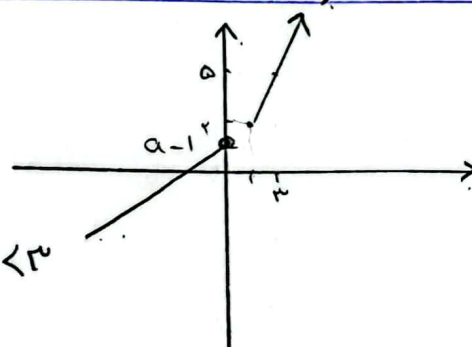


$f(x) = \begin{cases} 3x - 1 & x \geq 1 \\ ax + a - 1 & x \leq 0 \end{cases}$

$a > 0$

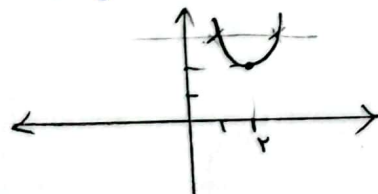
$R_{f_1} = (-\infty, a-1] \rightarrow a-1 < 2 \Rightarrow a < 3$

$\begin{cases} a < 3 \\ a > 0 \end{cases} \rightarrow 0 < a < 3$



الف) $y = x^3 + 2$ اثبات $y_1 = y_2 \rightarrow x_1^3 + 2 = x_2^3 + 2 \rightarrow x_1^3 = x_2^3 \rightarrow x_1 = x_2$
ضابطه وارون $y - 2 = x^3 \xrightarrow{\sqrt[3]{\quad}} x = \sqrt[3]{y-2} \rightarrow f^{-1}(y) = x = \sqrt[3]{y-2}$

ب) $y = (x^2 - 4x + 2) + 2 - 2 = x^2 - 4x + 2 = (x-2)^2 + 2$



طبق نمودار یک یک است
چون خط افقی نمودار را در دو نقطه
قطع می کند.

الف) $y = \frac{3x+1}{x-2} \rightarrow y_1 = y_2 \Rightarrow \frac{3x_1+1}{x_1-2} = \frac{3x_2+1}{x_2-2} \rightarrow (3x_1+1)(x_2-2) = (3x_2+1)(x_1-2)$

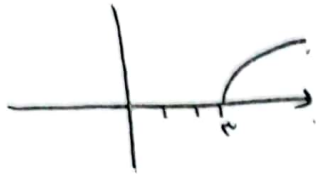
$3x_1x_2 - 6x_1 - x_2 + 2 = 3x_2x_1 - 6x_2 - x_1 + 2 \rightarrow -6x_1 + x_1 = -6x_2 + x_2 \rightarrow -5x_1 = -5x_2$

$\rightarrow x_1 = x_2$ ضابطه وارون $y(x-2) = 3x+1 \rightarrow xy - y = 3x+1 \Rightarrow xy - 3x = y+1$

$x(y-3) = y+1 \rightarrow x = \frac{y+1}{y-3} \rightarrow f^{-1}(y) = x = \frac{y+1}{y-3}$

ب) $y = \frac{x^2 + 2x}{x+2} = \frac{x(x+2)}{x+2} = x \rightarrow y = x$ کتاب یک به یک و یک به یک

الف) $y = \sqrt{x-3}$ $y^2 = x-3 \rightarrow y^2+3 = x \rightarrow f^{-1}(x) = y = x^2+3$ (9)



کتاب یک به یک

ب) $y = x^2 - 4x + 3$ $x \in [2, +\infty)$ $y = (x-2)^2 - 1 \rightarrow \sqrt{y+1} = |x-2|$
 $y = x^2 - 4x + 3 + 1 - 1 = (x-2)^2 - 1$ $\sqrt{y+1} = x-2 \rightarrow \sqrt{y+1} + 2 = x$
 $\rightarrow f^{-1}(x) = y = \sqrt{x-2} + 2$



$f^{-1}(x) = ax + b - \sqrt{x-c}$ (۷) واردين تابع $f(x) = x + \sqrt{x}$ را با z به صورت

$y = x + \sqrt{x} + \frac{1}{\epsilon} - \frac{1}{\epsilon} = (\sqrt{x} + \frac{1}{\epsilon})^2 / f^{-1}(x) = x + \frac{1}{\epsilon} - \sqrt{x + \frac{1}{\epsilon}}$ حاصل $a+2b+4c$

$y + \frac{1}{\epsilon} = (\sqrt{x} + \frac{1}{\epsilon})^2 \rightarrow \sqrt{y + \frac{1}{\epsilon}} = \sqrt{x} + \frac{1}{\epsilon} \rightarrow \sqrt{y + \frac{1}{\epsilon}} - \frac{1}{\epsilon} = \sqrt{x}$ توان

$y + \frac{1}{\epsilon} + \frac{1}{\epsilon} - \sqrt{y + \frac{1}{\epsilon}} = x \rightarrow y - \sqrt{y + \frac{1}{\epsilon}} + \frac{1}{\epsilon} = x \Rightarrow f^{-1}(x) = x - \sqrt{x + \frac{1}{\epsilon}} + \frac{1}{\epsilon}$
 $a=1, b=\frac{1}{\epsilon}, c=-\frac{1}{\epsilon}$

$y = \frac{x}{\sqrt{x^2-2}} \rightarrow y_1 = y_2 \Rightarrow \frac{x_1}{\sqrt{x_1^2-2}} = \frac{x_2}{\sqrt{x_2^2-2}} \xrightarrow{\text{توان}} \frac{x_1^2}{x_1^2-2} = \frac{x_2^2}{x_2^2-2}$ (۸)

$\rightarrow x_1^2 x_2^2 - x_1^2 = x_1^2 x_2^2 - x_2^2 \rightarrow x_1 = x_2$ کتاب یک به یک

$y = \frac{x}{\sqrt{x^2-2}} \rightarrow y^2 = \frac{x^2}{x^2-2} \Rightarrow x^2 y^2 - 2y^2 = x^2 \Rightarrow x^2 y^2 - x^2 = 2y^2 \rightarrow$

$x^2 (y^2 - 1) = 2y^2 \rightarrow x^2 = \frac{2y^2}{y^2-1} \rightarrow x = \pm \frac{\sqrt{2} y}{\sqrt{y^2-1}} \rightarrow f^{-1}(x) = y = \frac{\sqrt{2} x}{\sqrt{x^2-1}}$

$f^{-1}(x) = \frac{x}{1+|x|}, g(x) = \sqrt{x} - 1$ $f(-\frac{y}{a}) + g^{-1}(-\frac{y}{a}) = -\frac{y}{a} + \frac{a}{a} = \frac{-10+27}{10} = \frac{17}{10}$ (9)

$f(x) = \frac{x}{1-|x|} \rightarrow f(-\frac{y}{a}) = \frac{-\frac{y}{a}}{1-\frac{y}{a}} = \frac{-\frac{y}{a}}{\frac{a-y}{a}} = -\frac{y}{a-y}$

$g^{-1}(-\frac{y}{a}) \rightarrow -\frac{y}{a} = \sqrt{x} - 1 \rightarrow -\frac{y}{a} + 1 = \sqrt{x} \rightarrow \frac{y}{a} = \sqrt{x} \xrightarrow{\text{توان}} \frac{y^2}{a^2} = x$

$g^{-1}(-\frac{y}{a}) = \frac{y}{a}$

10. $f(x) = \sqrt{x-1}$ $\rightarrow f^{-1}(x) = ?$ $g(x) = f(x) + \sqrt{f(x)}$ $\rightarrow g^{-1}(12) = ?$

$$y = \sqrt{x-1} \xrightarrow{\text{square}} y^2 = x-1 \rightarrow y^2+1 = x \rightarrow y = \sqrt{x^2+1} \leftarrow f(x)$$

$$g(x) = \sqrt{x^2+1} + \sqrt{\sqrt{x^2+1}} + \frac{1}{x} - \frac{1}{x} = \left(\sqrt{x^2+1} + \frac{1}{x} \right)^2 - \frac{1}{x^2}$$

$$12 = \left(\sqrt{x^2+1} + \frac{1}{x} \right)^2 - \frac{1}{x^2} \rightarrow \sqrt{12 + \frac{1}{x^2}} = \sqrt{\left(\sqrt{x^2+1} + \frac{1}{x} \right)^2} \xrightarrow{\sqrt{\quad}} \pm \frac{\sqrt{12}}{x} = \sqrt{x^2+1} + \frac{1}{x}$$

$$\frac{\sqrt{12}}{x} - \frac{1}{x} = \left(\sqrt{x^2+1} \right)^2 \rightarrow 9 = x^2+1 \rightarrow x^2=8 \rightarrow x=2 \rightarrow g^{-1}(12)=2$$