ماهها میرمقتدایی

۱ ←

$$f(x)=\begin{cases} x^3-4 & x>a \\ 12x-20 & x\le a \end{cases} \Longrightarrow$$

$$x^3-4=12x-20 \rightarrow x^3-12x+16=0 \xrightarrow[\text{تجزیه}]{\text{نامساوی}} (a-2)^2(a+4)\geq 0$$

$$\Longrightarrow \boxed{a \geq -4}$$

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۲ ←

$$f(x)=3x+k \qquad f^{-1}(2)=4 \rightarrow f(4)=2$$

$$2=12+k \rightarrow k=-10$$

$$f(x)=3x-10 \rightarrow f(7)=21-10=\boxed{11}$$

الف

ب

$$f(f(x))=3(3x-10)-10=9x-30-10=\boxed{9x-40}$$

---

۳ ←

$$y=\frac{ax}{x-1} \rightarrow x=\frac{ay}{y-1} \rightarrow xy-x=ay$$

$$y(x-a)=x \rightarrow y=\frac{x}{x-a} \rightarrow$$

$$a = \frac{۲a}{۲a-a} = \boxed{a = ۲}$$

۴

$$g = \{(۳,۲),(۸,۶),(۹,۵)\} \rightarrow g^{-1} = \{(۲,۳),(۶,۸),(۵,۹)\}$$

$$f = \{(۳,۵),(۴,۷),(۷,۲),(۹,۶)\} \rightarrow f^{-1} = \{(۵,۳),(۷,۴),(۲,۷),(۶,۹)\}$$

الف

$$\boxed{f(f^{-1}(x)) = \{(۵,۵),(۷,۷),(۲,۲),(۶,۶)\}}$$

ب

$$\boxed{f^{-1}(f(x)) = \{(۳,۳),(۴,۴),(۷,۷),(۹,۹)\}}$$

ج

$$\boxed{f(g^{-1}(x)) = \{(۲,۵),(۵,۶)\}}$$

د

$$\boxed{g^{-1}(f(x)) = \{(۳,۹),(۷,۳),(۹,۸)\}}$$

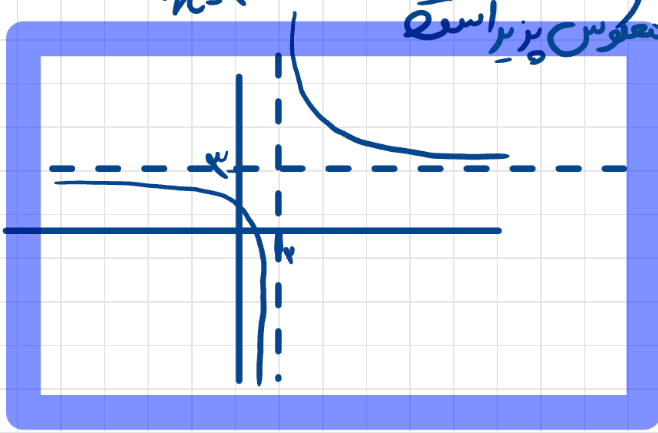
$f = \{(۲,۴),(۴,۸),(۶,۰)\}$ $h = \{(۱,۲),(۳,۴),(۴,۱),(۵,۱)\}$ ← ۵

$g = \{(۲,۱),(۴,۳),(۶,۵)\} \to g^{-1} = \{(۱,۲),(۳,۴),(۵,۶)\}$

$\dfrac{h}{f(g^{-1}(x))} \to \boxed{\{(۱,\frac{۱}{۲}),(۳,\frac{۱}{۲})\}}$

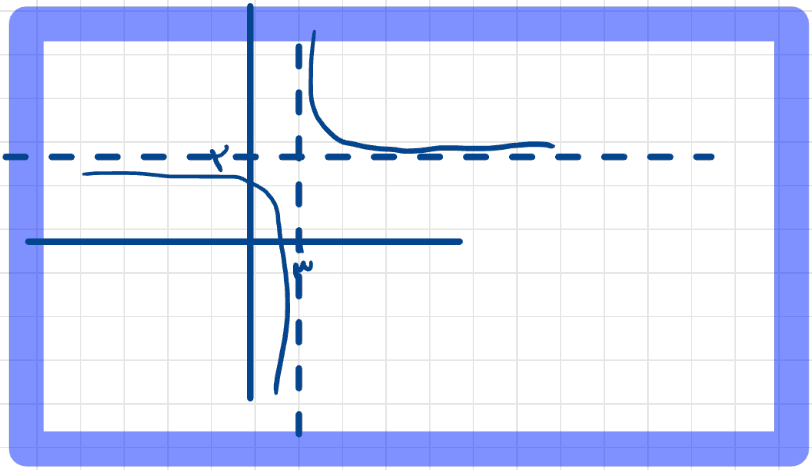
$f(g^{-1}(x)) \to \{(۱,۴),(۳,۸),(۵,۰)\}$

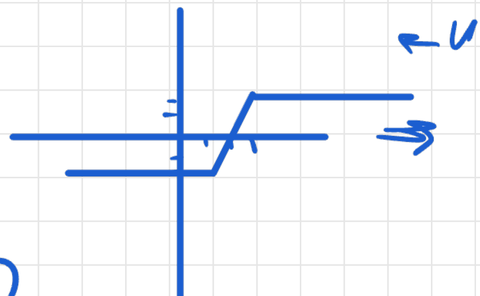
$y = \dfrac{۳x+۱}{x-۲} \to$ تابع هموگرافیه پس قطعاً یک‌به‌یک و درنتیجه معکوس‌پذیر است ← ۴



$\to x = \dfrac{۳y+۱}{y-۲} \to$

$y(x-۳) = ۲x+۱ \to \boxed{y = \dfrac{۲x+۱}{x-۳}}$



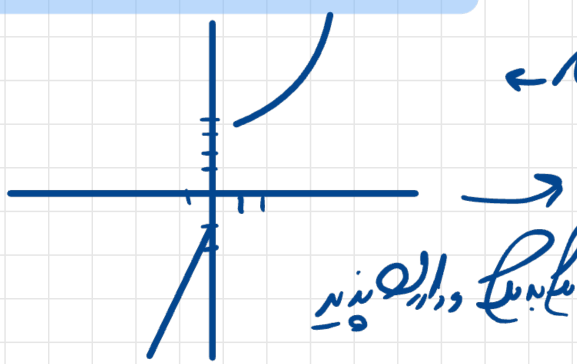


$$f(x)=\begin{cases} 2 & x\geqslant 3 \\ 2x-4 & 1\leqslant x\leqslant 3 \\ -2 & x\leqslant 1 \end{cases}$$

دربازه $[1,3]$ ← تابع یک به یک وارون پذیر است

$$y=2x-4 \Rightarrow x=2y-4 \rightarrow x+4=2y \Rightarrow \boxed{y^{-1}=\frac{x+4}{2}}$$

$$f(x)=\begin{cases} x^3+4 & x\geqslant 1 \\ 4x-1 & x\leqslant 0 \end{cases}$$



تابع یک به یک وارون پذیر

$$y = x^3 + 4 \rightarrow x \geqslant 1 \rightarrow R_f = [5, +\infty)$$

$$x = y^3 + 4 \rightarrow y \geqslant 5$$

$$y^3 = x - 4 \rightarrow y = \sqrt[3]{x-4} \quad , y \geqslant 5$$

$$y = 4x - 1 \rightarrow x \leqslant 0 \rightarrow R_f = (-\infty, -1]$$

$$x = 4y - 1 \rightarrow y \leqslant -1 \rightarrow 4y = x + 1 \rightarrow$$

$$y = \frac{x+1}{4} \quad , y \leqslant -1$$

$$\Longrightarrow \boxed{f^{-1}(x) = \begin{cases} \sqrt[3]{x-4} \quad , x \geqslant 5 \\ \dfrac{x+1}{4} \quad , x \leqslant -1 \end{cases}}$$

$$\leftarrow 9$$

$$f(x) = x^2 - \frac{(x+1)^3}{x+3} \rightarrow \frac{x^3 + \cancel{3x^2} - \cancel{x^3} - 1 - 3x + \cancel{3x^2}}{x+3} \rightarrow$$

$$(x+1)^3 = x^3 + 1 + 3x + 3x^2$$

$$y = \frac{-3x-1}{x+3} \rightarrow xy + 3y = -3x - 1 \rightarrow x(y+3) = -3y - 1$$

$$\rightarrow y^{-1} = \frac{-3x-1}{x+3} \xrightarrow[\times 2]{} y^{-1} = \frac{-6x-2}{2x+6} \rightarrow \begin{array}{l} a = -6 \\ b = -2 \\ d = 6 \end{array}$$

$$f^{-1}(-2) = ? \rightarrow f^{-1}(-2) = \frac{6-1}{-2+3} = \boxed{5}$$

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۱۰ $\leftarrow$ $y = \frac{x}{x^2+1} \to y(x^2+1) = x \to yx^2 - x + y = 0$

$yx^2 + y = x \to yx^2 + y - x = 0 \to$ ضمناً f روی R یک به یک است

برد $f = [-\frac{1}{2}, \frac{1}{2}]$ ، دامنه $= [-1,1]$ $\xleftarrow{f^{-1}}$ دامنه $= [-\frac{1}{2}, \frac{1}{2}]$ ، برد $= [-1,1]$

$$\left.\begin{array}{l} yx^2 + y - x = 0 \\ \Delta = 1 - 4y^2 \end{array}\right\} \to \frac{1 \pm \sqrt{1-4y^2}}{2y} = x \xrightarrow{y \neq 0}$$

$$\frac{1 \pm \sqrt{1-4x^2}}{2x} = y^{-1} \xrightarrow{x \neq 0} \boxed{f^{-1}(y) = \frac{2x}{1+\sqrt{1-4x^2}}, x \in [-\frac{1}{2}, \frac{1}{2}]}$$