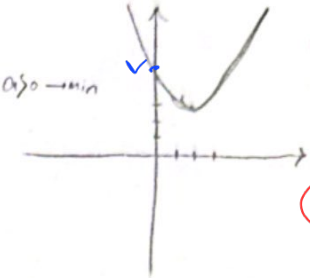
۱۹

۱. پارسا سامانی — یازدهم دختر A — تکلیف شماره ۱۲

-۱

الف)

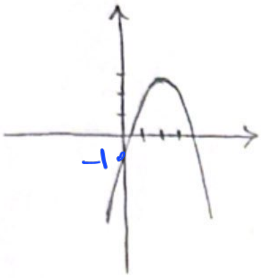


$a>0 \rightarrow min$

$S \left| \begin{array}{l} -\frac{b}{2a}=2 \\ 3 \end{array} \right.$

۱/۵

ب)



$S \left| \begin{array}{l} 2 \\ 3 \end{array} \right.$

-۲

الف) دو ریشه متمایز ← $\Delta>0 \Leftarrow$

$2x^2+(m+1)x+\frac{1}{2}m+2=0$

$(m+1)^2-4(2)(\frac{1}{2}m+2) = m^2-2m-15>0 \rightarrow 4-4(-15)(1)=64 \rightarrow \frac{2\pm\sqrt{64}}{2} \rightarrow 5,\ -3 \quad \frac{-3 \quad 5}{+\ \ 0\ -\ 0\ \ +} \rightarrow m\in(-\infty,-3)\cup(5,+\infty)$

$m^2+2m+1 \qquad -4m-16$

۲

ب) ریشه مضاعف ← $\Delta=0 \Leftarrow$ طبق حل بالا $m^2-2m-15=0 \Leftarrow$ $\boxed{m=\{-3,5\}}$

ج) معادله فاقد ریشه ← $\Delta<0 \Leftarrow$ طبق حل الف ← $m^2-2m-15<0$ $\quad \frac{-3 \quad 5}{+\ \ 0\ -\ 0\ \ +} \rightarrow \boxed{m\in(-3,5)}$

د) معادله ریشه داشته باشد ← $\Delta\geq 0$ ← $m^2-2m-15\geq 0$ $\quad \frac{-3 \quad 5}{+\ \ 0\ -\ 0\ \ +} \Rightarrow m\in(-\infty,-3]\cup[5,+\infty)$

-۳

الف) ریشه ها منفی $\Leftarrow$ $S<0$ و $\Delta>0$ و $P>0$

$y=(m-2)x^2-2(m+1)x+10$

$\Delta>0 \rightarrow (-2m-2)^2-4(m-2)(10) = 4m^2+8m+4-40m+80 \Rightarrow 4m^2-32m+84>0 \xrightarrow{\div 4} m^2-8m+21>0$

$10m-20 \qquad -40m+80$

معادله فاقد ریشه $64-4(21)<0$

Δ همواره مثبت است

$S<0 \rightarrow \frac{-b}{a}<0 \rightarrow \frac{2m+2}{m-2}<0 \quad \frac{-1 \quad 2}{+\ 0\ -\ \text{ن}\ +} \rightarrow m\in(-1,2)$ I

$P>0 \rightarrow \frac{c}{a}>0 \rightarrow \frac{10}{m-2}>0 \quad \frac{2}{-\ \text{ن}\ +} \rightarrow m\in(2,+\infty)$ II

$\boxed{\mathrm{I}\cap\mathrm{II}\ \varnothing}$

۵

ب) $\Delta>0$ (همواره برقرار) و $P<0$ و $S<0$ (قدر مطلق ریشه منفی کوچکتر است)

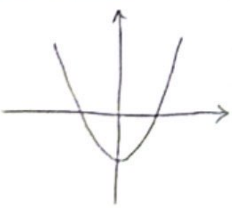
$\frac{c}{a}>0 \xrightarrow{\text{طبق الف}} \frac{2}{-\ \text{ن}\ +} \rightarrow m\in(-\infty,2)$ I

$-\frac{b}{a}<0 \xrightarrow{\text{طبق الف}} \frac{-1 \quad 2}{+\ 0\ -\ \text{ن}\ +} \rightarrow m\in(-1,2)$ II

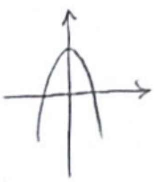
$\mathrm{I}\cap\mathrm{II}\ \boxed{m\in(-1,2)}$

-۴

الف) هر چهار ناحیه ← معادله دارای دو ریشه مختلف العلامت $\Leftarrow$ $\Delta>0$ و $P<0$



یا



طبق سؤال بالا همواره $\Delta>0$ است و a طبق شکل ها در دو حالت $a<0$ یا $a>0$ می تواند داشته باشد و S هم هر دو حالت را دارد ← فقط درباره P می توان اظهار نظر کرد

$P<0 \leftarrow \frac{c}{a}<0 \leftarrow \frac{10}{m-2}<0 \Leftarrow \frac{2}{-\ \text{ن}\ +}$ $\quad \boxed{m\in(-\infty,2) \ ,\ m<2}$

۵

ب)

$$\frac{-b}{2a} = -2 \Rightarrow \frac{2m+2}{2m-4} = -2 \Rightarrow -4m+8 = 2m+2 \to 6m = 6 \to \underline{m=1}$$

ه- دارای ریشه $\Leftarrow \Delta \geq 0$

$$\frac{2x^2}{3} - (2\sin\alpha)x + \frac{3}{2} = 0 \xrightarrow{\Delta \geq 0} 4\sin^2\alpha - 4\left(\cancel{\frac{2}{3}}\right)\left(\cancel{\frac{3}{2}}\right) \Rightarrow 4\sin^2\alpha - 4 \geq 0 \to 4\sin^2\alpha \geq 4 \Rightarrow \sin^2\alpha \geq 1 \Rightarrow \sin\alpha = 1\ \text{یا}\ -1$$

$$\text{if } \sin\alpha = -1 \to \frac{2x^2}{3} + 2x + \frac{3}{2} = 0 \to 4 - 4\left(\frac{2}{3}\right)\left(\frac{3}{2}\right) = 0 \to \frac{-2}{\frac{4}{3}} \Rightarrow x = -\frac{6}{4} = -\frac{3}{2}$$ غ ق ق

$$\text{if } \sin\alpha = 1 \to \frac{2x^2}{3} - 2x + \frac{3}{2} = 0 \to 4 - 4\left(\frac{2}{3}\right)\left(\frac{3}{2}\right) = 0 \to \frac{2}{\frac{4}{3}} = \frac{3}{2}\ \checkmark$$

۶-

$$3x^2 - 2x - 1 \xrightarrow{a+b+c=0} \Rightarrow x=1,\quad x = \frac{c}{a} = -\frac{1}{3}$$

$$2x^2 - 3x - 5 \xrightarrow{a+c=b} x=-1,\quad x = \frac{5}{2}$$

$$y = \frac{\cancel{(x-1)}^{-1}\left(x+\frac{1}{3}\right)\cancel{(x+1)}\left(x-\frac{5}{2}\right)}{\cancel{(1-x)}\cancel{(1+x)}} = -\left(x+\frac{1}{3}\right)\left(x-\frac{5}{2}\right) = \qquad x \neq \pm 1$$

$$y = -x^2 + \frac{13}{6}x + \frac{5}{6} \xrightarrow[y_{max} = \frac{-\Delta}{4a}]{a<0,\ max} \times +6 \to -6x^2 + 13x + 5 \to \frac{289}{24}$$

۷-

$$3S - 7 = 5P \qquad mx^2 - (m+2)x - 2 = 0 \qquad S = \alpha+\beta = \frac{m+2}{m},\quad P = \frac{-2}{m}$$

$$3\left(\frac{m+2}{m}\right) = 5\left(\frac{-2}{m}\right) + 7 \to$$

$$\frac{3m+6}{\cancel{m}} = \frac{-10+7m}{\cancel{m}} \to 3m+6 = -10+7m \to 4m = 16 \to m = 4 \to S = \frac{3}{2},\quad P = -\frac{1}{2} \qquad m \neq 0$$

$$(\alpha+\beta)^2 = S^2 - 2P \to \left(\frac{3}{2}\right)^2 - 2\left(-\frac{1}{2}\right) = \frac{13}{4}$$

۸-

$$x^2 - 5x + 2 = 0$$

$$S = 5 \to \alpha+\beta = 5 \to \alpha = 5-\beta$$

$$\beta^2 - 5\beta + 2 = 0 \to \beta^2 = 5\beta - 2 \to \beta^4 = (5\beta-2)^2 = 25\beta^2 - 20\beta + 4 \to 25(5\beta-2) - 20\beta + 4 = 0$$

$$125\beta - 50 - 20\beta + 4 = 0 \to \beta^4 = 105\beta - 46 \xrightarrow{\times\beta} \beta^5 = 105\beta^2 - 46\beta \to \beta^5 = 105\beta - 46 \xrightarrow{\times\beta} \beta^5 = 105\beta^2 - 46\beta$$

$$\beta^5 = 105(5\beta-2) - 46\beta \to \beta^5 = 479\beta - 210$$

$$\frac{4\alpha + \beta^5}{5\beta^2} \to \frac{4(5-\beta) + 479\beta - 210}{5(5\beta-2)} = \frac{475\beta - 190}{25\beta - 10} = \frac{19(25\beta-10)}{25\beta-10} = \underline{19}$$

$ax^2+bx+c \quad c=4$

$2x+10$

$\begin{cases} -(9a-3b+c=4) \\ 4a+2b+c=14 \end{cases} \quad \begin{cases} -5a+5b=10 \\ 9a-3b=0 \to 3a=b \end{cases}$

$2a=10, \; a=2.5$

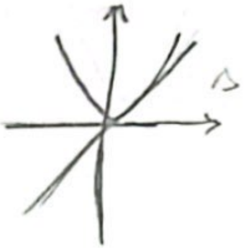
$b=7.5$

$\Rightarrow 2.5x^2+7.5x+4=0 \longrightarrow -\frac{b}{2a}=\frac{-7.5}{2\times 2.5}=-1.5$

5 -۹

-۱۰

$2x^2+(m+1)x+m+6=x \longrightarrow 2x^2+mx+m+6=0$



$\Delta=0$

$m^2-4(2)(m+6)=m^2-8m-48 \longrightarrow$

$\boxed{m=12 \text{ یا } -4}$

✗ ✓

1.5

$m=-4 \longrightarrow 2x^2-4x+2=0 \longrightarrow \Delta=0$ ✓

$m=12 \longrightarrow 2x^2+12x+18=0 \longrightarrow \Delta=0$ ✓