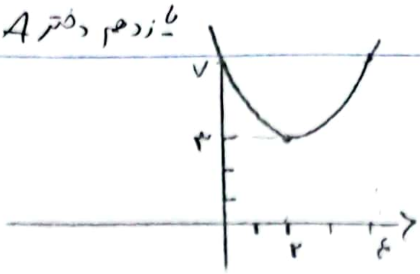
دیانا طبری — یازدهم دفتر A — تکلیف شماره ۱۲ — ۱۴,۲۵

۱)

الف) $y = x^2 - 4x + 7 = (x-2)^2 + 3$ ، $+4+3$

$S = (2,3)$

x	0	2	4
y	7	3	7

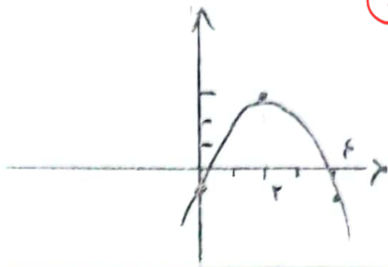


ب) $y = -x^2 + 4x - 1 = -(x^2 - 4x + 1) =$

$-(x^2 - 4x + 1 + 4 - 4) = -(x-2)^2 + 3$

$S(2,3)$

x	0	2	4
y	-1	3	-1



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۲)

$2x^2 + (m+1)x + \frac{1}{2}m + 2 = 0$

الف) $\Delta \geq 0 \Rightarrow (m+1)^2 - 4(2)(\frac{1}{2}m+2) = m^2 + 2m + 1 - 4m - 16 = m^2 - 2m - 15 = 0$

$(m-5)(m+3) = 0 \to \begin{cases} m = 5 \\ m = -3 \end{cases}$ $+ \; -3 \; - \; 5 \; +$ $(-\infty, -3] \cup [5, +\infty)$

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ب) $\Delta = 0$ ، $m = 5, -3$

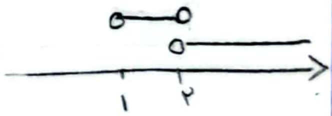
ج) $\Delta < 0 \to$ ج.م : $(-3, 5)$

د) $m \geq 0 \to (-\infty, -3] \cup [5, +\infty)$

۳)

$y = (m-2)x^2 - 2(m-1)x + 10$

الف) $\frac{c}{a} > 0 \to \frac{10}{m-2} > 0 \to m - 2 > 0 \to m > 2$



$-\frac{b}{a} < 0 \to \frac{2(m-1)}{m-2} < 0 \to \begin{cases} m = 1 \\ m = 2 \end{cases}$ $+ \; 1 \; - \; 2 \; +$ $1 < m < 2$

معادله جواب ندارد به ازای هیچ مقدار m رابطه برقرار نیست

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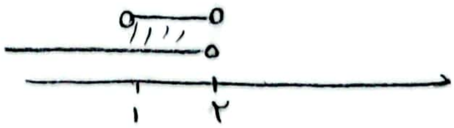
ب) $x_1 > 0$ ، $x_2 < 0$ $|x_2| > |x_1|$

$\frac{c}{a} < 0 \to m - 2 < 0 \to m < 2$



$-\frac{b}{a} < 0 \to 1 < m < 2$

ج.م : $1 < m < 2$

۴)

$y = (m-2)x^2 - 2(m+1)x + 10$ $P > 0 \to m < 2 \to m < 2$

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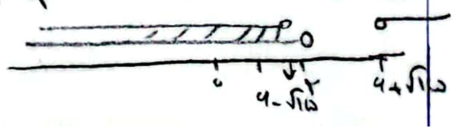
الف) $\Delta > 0 \to 4(m-1)^2 - 4(m-2)(10) = 4m^2 - 8m + 4 - 40m + 80 =$

$4m^2 - 48m + 84 = 0 \to 4(m^2 - 12m + 21) = 0 \to \Delta = 144 - 84 = 60 \to \sqrt{\Delta} = 2\sqrt{15}$

$m_1 = \frac{12 + 2\sqrt{15}}{2} = \frac{2(6+\sqrt{15})}{2}$ ، $m_2 = 6 - \sqrt{15}$ $+ \; 6-\sqrt{15} \; - \; 6+\sqrt{15} \; +$ $m < 6 - \sqrt{15}$, $m > 6 + \sqrt{15}$

و $\frac{c}{a} < 0 \to \frac{10}{m-2} < 0 \to m - 2 < 0 \to m < 2$



ج.م $= m < 6 - \sqrt{15}$

ب) $x_S = \frac{-b}{2a} = -2 \rightarrow \frac{x(m+1)}{x(m-2)} = -2 \Rightarrow m+1 = -2m+4 \Rightarrow$

$3m = 3 \rightarrow m = 1$

$\frac{2}{3}x^2 - (2\sin\alpha)x + \frac{3}{2} = 0 \qquad \Delta = 0, \ x > 0$

$\Delta = (2\sin\alpha)^2 - 4\left(\frac{2}{3}\right)\left(\frac{3}{2}\right) = 4\sin^2\alpha - 4 = 0 \rightarrow 4\sin^2\alpha = 4$

$\sin^2\alpha = 1 \rightarrow \sin\alpha = \pm 1 \longrightarrow \sin\alpha = 1$

$\frac{2}{3}x^2 - 2x + \frac{3}{2} = 0 \xrightarrow{\times 6} 4x^2 - 12x + 9 = 0 \rightarrow (2x-3)^2 = 0 \rightarrow 2x = 3 \rightarrow x = \frac{3}{2}$

$y = \frac{(3x^2 - 2x - 1)(2x^2 - 3x - 5)}{1 - x^2} = \frac{(3x+1)(x-1)(2x-5)(x+1)}{-(x-1)(x+1)}$

برای $x \neq -1, x \neq 1$:

$y = -[(3x+1)(2x-5)]$

$y = -(6x^2 - 15x + 2x - 5) = -6x^2 + 13x + 5 \qquad a = -6 < 0$

$x_S = \frac{-b}{2a} = \frac{-13}{-12} = \frac{13}{12} \neq 1, -1 \rightarrow y_S = -6\left(\frac{13}{12}\right)^2 + 13\left(\frac{13}{12}\right) + 5 = -6 \times \frac{169}{144} + \frac{169 \times 2}{12 \times 12} + \frac{5 \times 144}{144}$

$\frac{-1014 + 2028 + 720}{144} = \frac{1734}{144} = \frac{289}{24}$ بیشترین مقدار تابع

$mx^2 - (m+2)x - 2 = 0$

$S = x_1 + x_2 = \frac{m+2}{m}$

$P = x_1 \cdot x_2 = \frac{-2}{m}$

$m = 4 \qquad S^2 - 2P = \frac{13}{4}$

$\rightarrow 3S + 7 = 5P \Rightarrow 3S - 5P = -7 \Rightarrow 3\left(\frac{m+2}{m}\right) - 5\left(\frac{-2}{m}\right) = -7$

$\frac{3m+6}{m} + \frac{10}{m} = -7 \Rightarrow \frac{3m+16}{m} = -7 \rightarrow 3m + 16 = -7m \Rightarrow$

$10m = -16 \rightarrow m = \frac{-16}{10} = \frac{-8}{5} \Rightarrow -\frac{8}{5}x^2 - \left(\frac{-8}{5} + 2\right)x - 2 = 0$

$\rightarrow -\frac{8}{5}x^2 - \frac{2}{5}x - 2 = 0 \longrightarrow x_1^2 + x_2^2 = S^2 - 2P = \frac{1}{16} - 2\left(\frac{5}{4}\right) = \frac{1-40}{16} = \frac{-39}{16}$

$S = -\frac{\frac{-2}{5}}{\frac{-8}{5}} = -\frac{2}{8} = \frac{1}{4}, \quad P = \frac{-2}{-\frac{8}{5}} = \frac{10}{8} = \frac{5}{4}$

$x^2 - 5x + 2 = 0$

$\beta^2 = 5\beta - 2 \qquad (\beta^2)^2 = (5\beta - 2)^2 = 25\beta^2 - 20\beta + 4 \rightarrow \beta^4 = 25\beta^2 - 20\beta^2 + 4\beta$

$\alpha + \beta = 5 \rightarrow \alpha = 5 - \beta$

$\frac{4\alpha + \beta^4}{5\beta^2} = \frac{4(5-\beta) + 25\beta^2 - 20\beta^2 + 4\beta}{25\beta - 10} = \frac{25\beta^2 - 20\beta^2 + 20}{25\beta - 10}$

$\frac{\alpha^2 + \beta^2}{5} = \frac{S^2 - 2P}{5}$

$= 19$

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$y_1 = ax^2 + bx + c \quad , \quad y_2 = 2x + 10$

$x_1 = 2 \rightarrow 2(2)+10 = 14 \rightarrow (2,14) \xrightarrow{y_1} 14 = a(2)^2 + b(2) + c \rightarrow 4a + 2b + 4 = 14$

$x_2 = -3 \rightarrow 2(-3)+10 = 4 \rightarrow (-3,4) \xrightarrow{y_1} 4 = a(-3)^2 + b(-3) + c \rightarrow 9a - 3b + 4 = 4$

$(0,4) \quad (0,4) \rightarrow c = 4$

$\begin{cases} 4a + 2b = 10 \\ 9a - 3b = 0 \end{cases} \rightarrow \begin{cases} 12a + 6b = 30 \\ 18a - 6b = 0 \end{cases}$

$30a = 30 \rightarrow a = 1 \quad b = 3 \quad \rightarrow y = x^2 + 3x + 4$

محور تقارن $x = x_S = -\frac{3}{2}$

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$y = 2x^2 + (m+1)x + m + 9 \qquad (x_S, y_S) \rightarrow x_S = y_S$

$\Delta = 0 \rightarrow m = 12, -4$ ✓ ✗

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$x_S = \frac{-b}{2a} = \frac{-(m+1)}{4}$

$-\frac{(m+1)}{4} = \frac{2(-(m+1))^2}{16} - \frac{(m+1)(m+1)}{4} + m + 9 = \frac{m^2+2m+1}{8} - \frac{m^2+2m+1}{4} + m + 9$

$\frac{m^2 + 2m + 1 - 2m^2 - 4m - 2 + 8m + 72}{8} = \frac{-m^2 + 6m + 71}{8}$

$-\frac{(m+1)}{4} = \frac{-m^2 + 14m + 68}{8} \rightarrow -8m - 8 = -4m^2 + 56m + 192$

$-4m^2 + 56m + 192 + 8m + 8 = 0$