تکلیف شماره ۱۳ — یازدهم دختر A — سانیا شیدمحمدی

①

$y=(a-1)x^2+x+3$

$x=2$ ← متقارن $\Rightarrow x_S=2 \Rightarrow \frac{-b}{2a}=\frac{-1}{2a-2}=2 \Rightarrow 4a-4=-1 \Rightarrow 4a=3 \Rightarrow a=\frac{3}{4}$

$\Rightarrow y=-\frac{x^2}{4}+x+3 \Rightarrow -\frac{x^2}{4}+x+3=0 \xrightarrow{\times -4} x^2-4x-12=0 \Rightarrow (x-6)(x+2)=0$

$x=\frac{1}{2},\ x=\frac{-3}{2} \Leftarrow x=\frac{6}{-4},\ x=\frac{-2}{-4}$

$x=\frac{1}{2}$

②

$f(x)=mx^2-5x+m$

$m\in (I)\cap(II)=-\frac{5}{2}<m<0$

x		x_1		x_2	
$f(x)$	$-$	0	$+$	0	$-$

→ هم علامت a

هم علامت a — خلاف علامت a

$\Rightarrow m<0\ (I)$

$, \Delta>0 \Rightarrow 25-4(m)(m)=25-4m^2>0$

$m^2=\frac{25}{4}$

$4m^2-25<0 \Rightarrow$

$-\frac{5}{2}$		$\frac{5}{2}$	
$+$ 0	$-$	0	$+$

$\Rightarrow -\frac{5}{2}<m<\frac{5}{2}\ (II)$

③

$\alpha=\frac{3-\sqrt5}{3} \quad \beta=\frac{3+\sqrt5}{3} \Rightarrow \Delta=5,\ -b=3,\ 3=2a$

$9-4(\frac{3}{2})(c)=5 \Rightarrow 9-6c=5 \Rightarrow 6c=4 \Rightarrow c=\frac{4}{6}$

$\Rightarrow y=\frac{3}{2}x^2-3x+\frac{2}{3}$

$\boxed{y=9x^2-18x+4}$

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$\alpha=\frac{18-6\sqrt5}{18},\ \beta=\frac{18+6\sqrt5}{18} \Rightarrow \Delta=180 \quad b=-18,\ a=9$

$\Rightarrow 324-4(9)(c)=180 \Rightarrow 324-180=36c \Rightarrow 144=36c$

$\Rightarrow c=4$

④

$2x^2-8x+m+2=0 \qquad \alpha<\beta \qquad 3\alpha^2+\beta^2=12$

$\alpha+\beta=4 \Rightarrow \alpha=4-\beta \Rightarrow \alpha=1$

$3(4-\beta)^2+\beta^2=12$

$\alpha\cdot\beta=\frac{m+2}{2} \Rightarrow \frac{m+2}{2}=3$

$3(16+\beta^2-8\beta)+\beta^2=12$

$\beta=3$

$48+3\beta^2-24\beta+\beta^2-12=0$

$m+2=6$

$m=4$

$\beta^2-6\beta+9=0 \Rightarrow (\beta-3)^2=0 \Leftarrow 4\beta^2-24\beta+36=0$

⑤

$c=5 \qquad S(2,9)$

$y=ax^2+bx+c$, $c=5$

$\frac{-b}{2a}=2 \Rightarrow -b=4a \Rightarrow b=-4a$

$x=2$: $4a-8a+5=9$

$\Rightarrow y=-x^2+4x+5$

$-4a+5=9 \Rightarrow a=-1,\ b=4$

ریشه‌ها $\Rightarrow x^2-4x-5=(x-5)(x+1)$

$\Rightarrow x=5,\ -1$

-1		5	
$-$ 0	$+$	0	$-$

$\boxed{(-1,5)}$

(۶)

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-7}{6}$

$\alpha x^2-7x+9\beta=0$

$\beta>0$ $\quad \frac{1}{\alpha}+\frac{1}{\beta}=?$

$\alpha\cdot\beta=\frac{9\beta}{\alpha} \Rightarrow \alpha^2=9 \Rightarrow \alpha=-3$ $\quad \alpha+\beta=\frac{2-9}{3}=\frac{7}{3}$

$\frac{7}{-3}=-3+\beta \Rightarrow 7=9-3\beta$ $\quad \beta=\frac{2}{3}$ $\quad \alpha\cdot\beta=\frac{-6}{3}$

$\alpha\beta^2-7\beta+9\beta=0 \Rightarrow 2\beta=-\alpha\beta^2 \Rightarrow \alpha<0$

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$x^2+mx-2m=0$ $\quad \alpha^2-m\beta=8$ $\quad \frac{-b}{a}=\alpha+\beta=?=-2$

$\alpha\cdot\beta=-2m \Rightarrow m\beta=-\beta^2+2m$

$\alpha+\beta=m$

if, $m=2 \Rightarrow x^2+2x-4=y$

if $\to m=-4 \Rightarrow x^2-4x+8$

$\Delta=16-4(1)(8)<0 \Rightarrow$ غ ق ق

چون دو ریشه دارد

$\alpha^2-m\beta=\underbrace{\alpha^2+\beta^2}_{S^2-2P}-2m=8$

m^2+4m

$\Rightarrow m^2+2m-8=0$

$(m+4)(m-2)=0 \Rightarrow m=-4,2$

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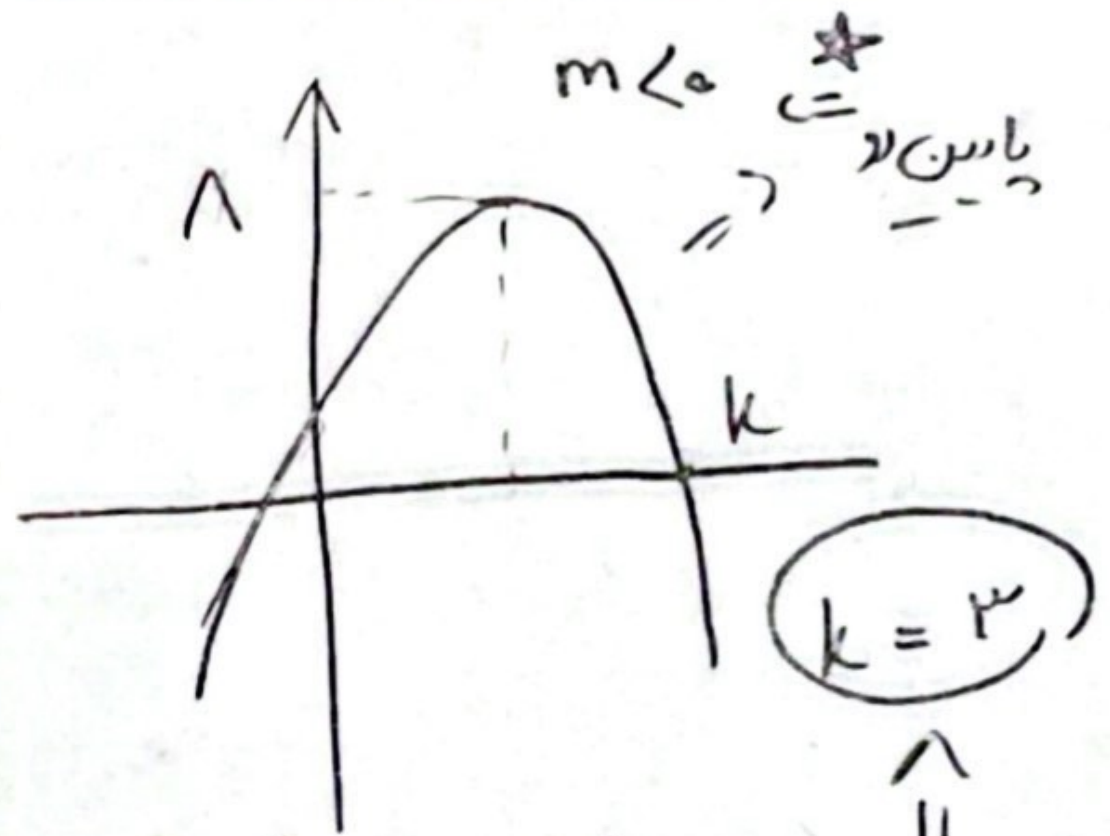

$m<0$ ⭑ $\Rightarrow$ پاسخ ما

$f(x)=mx^2+4x+\frac{m}{2}+7$

$y_S=8 \Rightarrow \frac{-\Delta}{4a}=8$

$\Delta=16-4(m)(\frac{m}{2}+7)=16-2m^2-28m$

$\Rightarrow \frac{2m^2+28m-16}{4m}=8 \Rightarrow 2m^2-4m-16=0$

$(m-8)(m+4) \Rightarrow m=4 \text{ و } -2$

$m=-2 \Leftarrow$ ⭑ با توجه به

$k=3 \Leftarrow 8$

$-2x^2+4x+6=0$

$\Rightarrow (x+6)(x-2) \Rightarrow x=3,\ x=-1$

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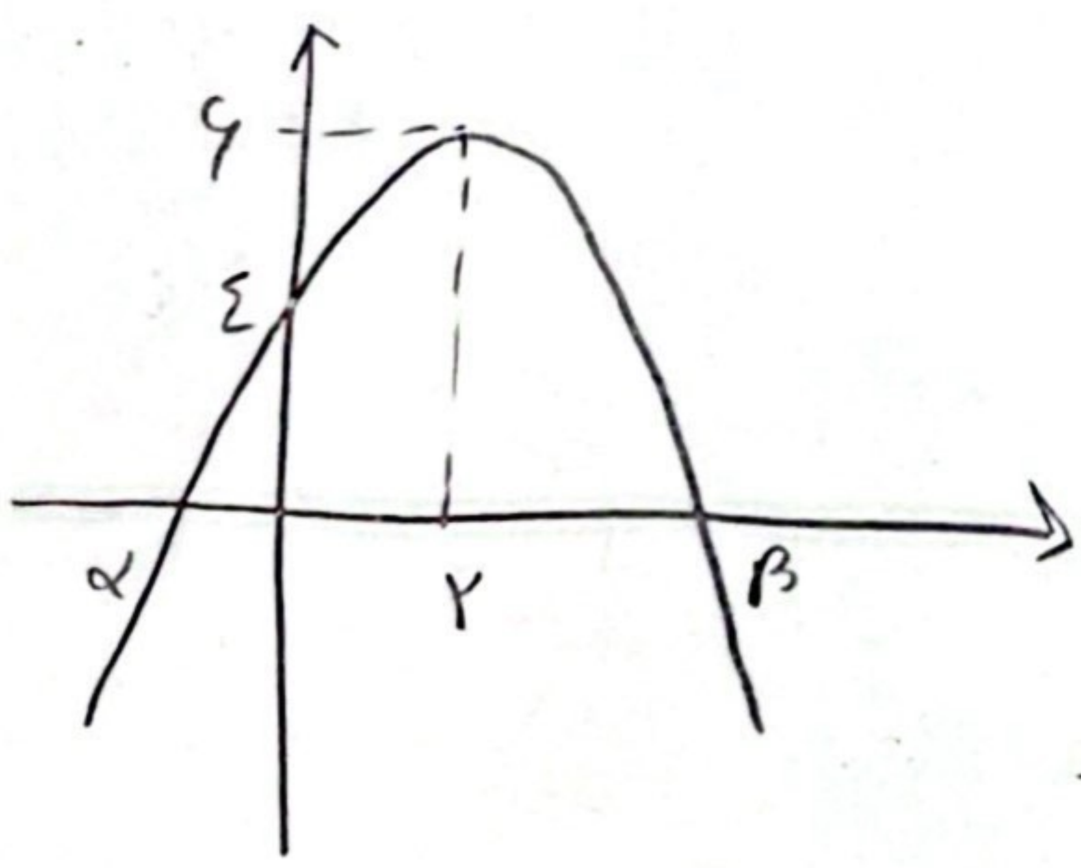
(۹)



$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\cdot\beta}=\frac{4}{-8}=\frac{-1}{2}$

$y=a(x-x_S)^2+y_S \Rightarrow y=a(x-2)^2+6$

$4=4a+6 \Rightarrow 4a=-2 \Rightarrow a=\frac{-1}{2}$

$\Rightarrow y=-\frac{1}{2}\overbrace{(x-2)^2}^{x^2+4-4x}+6$

$y=-\frac{x^2}{2}-2+2x+6=-\frac{x^2}{2}+2x+4 \Rightarrow \alpha+\beta=4,\ \alpha\cdot\beta=-8$

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(۱۰)

$x^2-(4a+4)x+(3a^2+6a+3)=0$

$\Rightarrow$ if $\to a=1 \Rightarrow x^2-8x+12$

$\Rightarrow (x-6)(x-2) \Rightarrow x=6,2$

$\cancel{4}-\cancel{8a}+\cancel{8}+3a^2+\cancel{6a}+\cancel{3}=0$

$\Downarrow$

$3a^2-2a-1=0 \Rightarrow a^2-2a-3=0=(a-3)(a+1)$

$\Rightarrow a=1,\frac{-1}{3}$

if $\to a=\frac{-1}{3} \Rightarrow x^2-\frac{8}{3}x+\frac{4}{3}$

$\Delta=\frac{64}{9}-4(1)(\frac{4}{3})=\frac{16}{9}$

$\Rightarrow x=\frac{\frac{8}{3}+\frac{4}{3}}{2}=2$

$x=\frac{\frac{8}{3}-\frac{4}{3}}{2}=\frac{2}{3}$