

$$y = (a-1)x^2 + x + 3$$

چون $x=2$ نسبت به نقطه $\rightarrow x_5 = 2 \rightarrow \frac{-b}{2a} = 2$

$$\frac{-b}{2a} = \frac{-1}{2a-2} = 2 \rightarrow 4a-4 = -1 \rightarrow a = \frac{3}{4}$$

$$y = -\frac{1}{4}x^2 + x + 3 \xrightarrow{\times 4} y = x^2 - 4x - 12 \rightarrow y = (x-4)(x+2) \rightarrow \boxed{x_1 = 4} \rightarrow x_2 = -2$$

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$$\begin{array}{c|cc} x & x_1 & x_2 \\ f(x) & - & + \end{array}$$

چون بین ۲ ریشه مخالف ضریب x^2 است پس ضریب x^2 منفی بوده که
بین ۲ ریشه مثبت شده $\leftarrow m < 0$ ①

$$\Delta > 0 \rightarrow \Delta = b^2 - 4ac \rightarrow 2\Delta - 4(m)(m) > 0 \rightarrow 4m^2 < 2\Delta$$

$$m^2 < \frac{2\Delta}{4} \rightarrow \frac{-\Delta}{4} < m < \frac{\Delta}{4} \quad \textcircled{2} \quad \textcircled{1} \cap \textcircled{2} = m \in \left(-\frac{\Delta}{4}, \frac{\Delta}{4}\right)$$

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$$y = x^2 - 5x + p$$

$$p = x_1 x_2 = \frac{3-\sqrt{5}}{3} \times \frac{3+\sqrt{5}}{3} = \frac{4}{9} \rightarrow y = x^2 - 2x + \frac{4}{9}$$

$$S = x_1 + x_2 = 3 - \frac{\sqrt{5} + 3 + \sqrt{5}}{3} = 2$$

ضرایب صحیح

$$\boxed{y = 9x^2 - 18x + 4}$$

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$$3\alpha^2 + \beta^2 = 12 \rightarrow 2\alpha^2 + \alpha^2 + 2\beta^2 - \beta^2 = 12 \rightarrow 2(\alpha^2 + \beta^2) + \alpha^2 - \beta^2 = 12$$

$$S=4 \quad p=\frac{m+2}{4} \rightarrow 2(14-m-2) + \left(\frac{4}{4}\right)\left(\frac{\sqrt{44-1(m+2)}}{4}\right) = 12 \xrightarrow{\div 2} (14-m) + \frac{\sqrt{44-1(m+2)}}{4} = 4$$

$$\sqrt{44-1(m+2)} = 4m-14 \xrightarrow{(\quad)^2} 44-1(m+2) = 16m^2 - 14m + 49$$

$$m^2 - 14m + 14 = 0 \rightarrow \boxed{m=4}$$

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$$y = a(x-h)^2 + k \xrightarrow{\text{ext } \left| \begin{smallmatrix} 2 \\ 9 \end{smallmatrix} \right.} y = a(x-2)^2 + 9 \xrightarrow{(0,5)} 5 = 4a + 9$$

$$\rightarrow a = -1 \rightarrow y = -x^2 + 4x + 5 \quad \frac{-x^2 + 4x + 5}{x = -1, 5} > 0 \quad \begin{array}{c} -1 \quad 5 \\ | \quad | \\ - \quad + \end{array}$$

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در بازه (۵، -۱) سی در بالای محور x ها قرار دارد

$$S = \frac{-b}{2a} = \frac{V}{\alpha} = \alpha + \beta \quad P = \alpha\beta = \frac{9\beta}{\alpha} \rightarrow \alpha^2 = 9\beta \rightarrow \alpha = \pm 3$$

$$\begin{aligned} \alpha = +3 &\rightarrow \frac{V}{\alpha} = \alpha + \beta \rightarrow \beta = \frac{-2}{3} \quad \text{O.O.E.} \\ \alpha = -3 &\rightarrow \frac{-V}{\alpha} = -\alpha + \beta \rightarrow \boxed{\beta = \frac{2}{3}} \quad \checkmark \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \boxed{\alpha = -3}$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{-1}{3} + \frac{2}{3} = \frac{V}{4}$$

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$$\alpha^2 = -m\alpha + 2m \rightarrow \alpha^2 - m\beta = \Lambda \rightarrow -m\alpha + 2m - m\beta = \Lambda$$

$$S = -m \quad P = -2m$$

$$-m(\alpha + \beta - 2) = \Lambda \rightarrow m^2 + 2m - \Lambda = 0 \quad \left. \begin{array}{l} \rightarrow m = 2 \\ \rightarrow m = -4 \end{array} \right\} \text{O.O.E.}$$

$$m = -4 \rightarrow \alpha^2 - 4\alpha + \Lambda = 0 \rightarrow \Delta < 0 \quad \text{O.O.E.}$$

$$m = 2 \rightarrow \alpha^2 + 2\alpha - 4 \rightarrow \Delta > 0 \quad \checkmark$$

$$\alpha + \beta = \frac{-b}{a} = \frac{-(4)}{1} = -4$$

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$$\text{ext} \left| \begin{array}{l} \frac{-b}{2a} = \frac{-r}{2m} = \frac{-r}{m} \\ r \end{array} \right\} \xrightarrow{x = \frac{-r}{m}} m\left(\frac{r}{m}\right) + r\left(\frac{-r}{m}\right) + \frac{m}{r} + V = \Lambda$$

$$\frac{\frac{r}{m} - \frac{\Lambda}{m} + \frac{m}{r}}{\frac{-r}{m}} = 1 \quad \frac{-\Lambda + m^2}{r m} = 1 \rightarrow m^2 - 2m - \Lambda = 0 \quad \left. \begin{array}{l} \rightarrow m = 4 \quad \text{O.O.E.} \\ \rightarrow m = -2 \end{array} \right\}$$

$$m = 4 \rightarrow 4\alpha^2 + 4\alpha + 9 \rightarrow \Delta < 0 \quad \text{O.O.E.} \quad m = -2 \rightarrow -2\alpha^2 + 4\alpha + 4 \rightarrow \Delta > 0 \quad \checkmark$$

$$f(x) = -2\alpha^2 + 4\alpha + 4 \rightarrow -2\alpha^2 + 4\alpha + 4 = 0 \quad \left. \begin{array}{l} \rightarrow \alpha = -1 \\ \rightarrow \alpha = 3 \end{array} \right\} \xrightarrow{K > 0} \boxed{K = 3}$$

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$$\text{ext} \left| \begin{array}{l} r \\ 4 \end{array} \right. \quad y = a(n-h)^2 + k \rightarrow y = a(n-r)^2 + 4 \xrightarrow{6 \div 4} r = 6a + 4 \rightarrow a = \frac{-1}{4}$$

$$y = \frac{-n^2}{4} + 2n + 4 \rightarrow S = 4 \quad P = -\Lambda$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{S}{P} = \frac{4}{-\Lambda} = \frac{-1}{4}$$

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$$n = 2 \xrightarrow{\text{در مساوی}} r - \Lambda a - \Lambda + 3a^2 + 4a + 3 = 0 \rightarrow 3a^2 - 2a - 1 = 0 \quad \left. \begin{array}{l} \rightarrow a = 1 \\ \rightarrow a = -\frac{1}{3} \end{array} \right\}$$

$$a = 1 \rightarrow n^2 - \Lambda n + 1P = 0 \quad \left. \begin{array}{l} \rightarrow n = 2 \\ \rightarrow n = 4 \end{array} \right\}$$

$$(n-4)(n-2) = 0$$

$$a = -\frac{1}{3} \rightarrow n^2 - \frac{\Lambda}{3}n + \frac{4}{3} = 0 \quad \left. \begin{array}{l} \rightarrow n = 2 \\ \rightarrow n = \frac{4}{3} \end{array} \right\}$$

$$(n^2 - 2)(n - \frac{4}{3}) = 0$$

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