

19,5

Arithmetical

جواب

تکلیف ۱۳، ۱۴

Arithmetical sequence $y = (a-1)n^2 + n + 1$

①

$$\frac{-b}{2a} = \frac{-1}{2(a-1)} = 1 \rightarrow \frac{1}{2(a-1)} = -1 \rightarrow \frac{1}{a-1} = -2 \rightarrow a = \frac{1}{-2} = -\frac{1}{2}$$

$$y = -\frac{1}{2}n^2 + n + 1$$

②

$$-\frac{1}{2}n^2 + n + 1 = 0$$

$$n = \frac{-1 \pm \sqrt{1+2}}{-\frac{1}{2}} \rightarrow n = \frac{-1+2}{-\frac{1}{2}} = -2$$

$$n = \frac{-1-2}{-\frac{1}{2}} = 6 \quad \text{جواب}$$

$$f(n) = mn^2 - an + m$$

③

n	x1	x2
f(n)	-a +	a -

$$\Delta > 0 \rightarrow (a-m)^2 > 0 \rightarrow (a-m)^2 > 0 \rightarrow \frac{(a-m)^2}{4} > 0$$

$$\frac{-a}{2} < m < \frac{a}{2} \quad ①$$

$$a < 0 \rightarrow m < 0 \quad ②$$

④

$$① \cap ② \Rightarrow \frac{-a}{2} < m < 0 \rightarrow m \in \left(\frac{-a}{2}, 0\right)$$

$$\left(x - \frac{r-\sqrt{a}}{r}\right) \left(x - \frac{r+\sqrt{a}}{r}\right) = \quad (2)$$

$$\frac{x^2 - \frac{r-r}{r}x + \left(\frac{-\frac{r-\sqrt{a}}{r} \times -\frac{r+\sqrt{a}}{r}\right)}{r} = x^2 - 1x + \frac{r}{r}$$

$$\frac{9-a}{9} = \frac{r}{9}$$

$\times 9$ ضرب في 9

$$rx^2 - 1x + m + r = 0$$

$$\alpha < \beta, \quad r\alpha^2 + \beta^2 = 12 \quad (3)$$

$$\alpha + \beta = 2 \rightarrow \beta = 2 - \alpha$$

$$r\alpha^2 + (2-\alpha)^2 = 12$$

$$r\alpha^2 + 14 + \alpha^2 - 4\alpha = 12$$

$$p = \frac{m+r}{r}$$

$$s = \frac{1}{r} = 2$$

$$r\alpha^2 - 4\alpha + 2 = 0$$

$$r\alpha^2 - 4\alpha + 14 = 0$$

$$\left(\alpha - \frac{2}{r}\right)^2 = 0 \rightarrow \alpha = 1 \Rightarrow$$

$$\beta = 1$$

$$p = \frac{m+r}{r} = 2 \rightarrow m+r=4 \rightarrow m=r$$



$$\frac{-b}{2a} = 1 \rightarrow -b = 2a \rightarrow b = -2a$$

$$ax^2 + bx + c = y$$

$$y = ax^2 - 2ax + c \quad (4)$$

$$9 = r\alpha - 1\alpha + c \rightarrow 9 - c = -1\alpha \rightarrow 2 = -1\alpha \rightarrow \alpha = -2$$

$$y = -x^2 + 2x + c$$

$$\begin{cases} x = -1 \\ x = 0 \end{cases}$$

$$\begin{array}{c|c|c} -1 & 0 & (-1, c) \\ \hline - & + & - \end{array}$$

$$\alpha^2 - Vn + 9\beta = 0$$

(4)

$$\frac{1}{\alpha} + \frac{1}{\beta} = \beta$$

$$p = \frac{9\beta}{\alpha} = \alpha\beta \rightarrow \alpha^2 = 9 \rightarrow \alpha = \pm 3 \rightarrow \alpha^2 - Vn + 9\beta = 0$$

$$S = \alpha + \beta = \frac{V}{3} \rightarrow \beta = \frac{V}{3} - \alpha \rightarrow \beta = -\frac{V}{3} \text{ or } \beta = \frac{V}{3}$$

(5)

$$\alpha = -3 \rightarrow -\alpha^2 - Vn + 9\beta = 0$$

$$S = \alpha + \beta = \frac{V}{3} \rightarrow \beta = \frac{V}{3} - \alpha = \frac{V}{3} + 3 \rightarrow \beta = \frac{V}{3} + 3$$

$$\frac{-1}{-3} + \frac{3}{1} = \frac{-1+9}{4} = \frac{8}{4} = 2$$

$$\alpha^2 + m\alpha - P_m = 0$$

$$\alpha + \beta = ?$$

(6)

$$\alpha^2 - m\beta = \Lambda \rightarrow \alpha^2 + \beta^2 - P_m = \Lambda$$

$$\beta^2 + \beta m - P_m = 0 \quad (\alpha + \beta)^2 - P_m = P_m + \Lambda$$

$$-m\beta = -\beta^2 + P_m \quad (-m)^2 + P_m = P_m + \Lambda$$

$$m^2 + P_m - \Lambda = 0$$

(7)

$$S = -m$$

$$P = -P_m$$

$$(m+2)(m-1) = 0 \rightarrow m = 1$$

$$m = -2$$

$$m = 1 \rightarrow \alpha^2 + P_m - \Lambda = 0$$

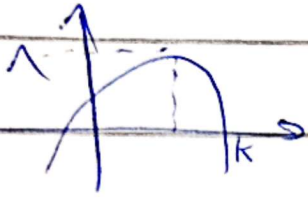
$$\Delta = 1 + 14 = 15 > 0 \text{ or } \Delta < 0$$

$$m = -2 \rightarrow \alpha^2 - P_m + \Lambda = 0$$

$$\Delta = 14 - 14 = 0 \text{ or } \Delta < 0$$

$$\alpha + \beta = -m = -1$$

$$f(x) = mx^2 + px + \frac{m}{p} + v \quad (1)$$



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$$\frac{-\Delta}{f_m} = \Delta \rightarrow \frac{-(14 - f_m(\frac{m}{p} + v))}{f_m} = \frac{f_m^2 + 14m - 14}{f_m} = \Delta$$

$$f_m^2 + 14m - 14 = 14f_m \rightarrow f_m^2 - f_m - 14 = 0$$

$$m^2 - f_m - 14 = 0$$

$$\left(\frac{m-1}{1}\right)\left(\frac{m+2}{1}\right) \Rightarrow m = -1$$

$\downarrow$
 $m = 2$

$$m = 2 \rightarrow y = 2x^2 + 8x + 9 \quad \Delta = 14 - 128 < 0 \quad \text{no real roots}$$

$$m = -1 \rightarrow y = -x^2 + px + 4$$

$$m^2 + px - 14 = 0$$

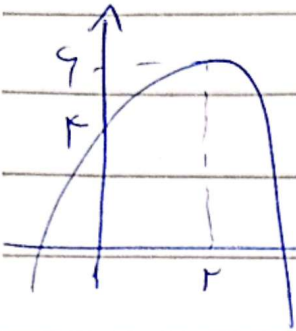
$$\left(\frac{x+4}{-1}\right)\left(\frac{x-2}{-1}\right) \Rightarrow x = -1$$

$$x = 3$$

Sum of roots =

$$k = 3$$

9



(1, 4) (2, 4)

$$y = ax^2 + bx + c \xrightarrow{(1, 4)}$$

$$y = ax^2 + bx + c$$

$$\frac{-b}{2a} = 2 \rightarrow -b = 4a$$

$$y = ax^2 - 4ax + c \xrightarrow{(2, 4)}$$

$$4 = 4a - 16a + c \rightarrow 4a = -12 \rightarrow a = -\frac{1}{3}$$

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$$y = -\frac{1}{3}x^2 + 2x + c$$

$$\frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab} = \frac{c}{-1} = \frac{-1}{2}$$

$$x^2 - (2a+2)x + (4a^2+4a+2) = 0$$

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$$\begin{cases} x=2 \\ x=? \end{cases}$$

$$8 = 4a + 2 \rightarrow 4 + 2 = 4a + 2 \rightarrow 2 = 4a \rightarrow a = \frac{1}{2}$$

$$8 = 4a^2 + 4a + 2 \rightarrow 4a^2 + 4a + 2 = 8 \rightarrow 4a^2 + 4a - 6 = 0 \rightarrow 2a^2 + 2a - 3 = 0$$

$$4a + 2 = 4a^2 + 4a + 2$$

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$$a=1 \rightarrow x^2 - 4x + 12 = 0$$

$$\Delta = 16 - 48 = -32$$

$$x = \frac{4 \pm \sqrt{-32}}{2} \rightarrow \boxed{x=2}$$

$$4a^2 - 4a - 1 = 0$$

$$a^2 - a - \frac{1}{4} = 0$$

$$\left(a - \frac{1}{2}\right)\left(a + \frac{1}{2}\right) = 0 \rightarrow \begin{cases} a = \frac{1}{2} \\ a = -\frac{1}{2} \end{cases}$$

$$a = -\frac{1}{2} \rightarrow x^2 - \frac{1}{2}x + \frac{1}{4} = 0 \rightarrow \Delta = \frac{1}{4} - 1 = -\frac{3}{4}$$

$$x = \frac{\frac{1}{2} \pm \sqrt{-\frac{3}{4}}}{2} \rightarrow \boxed{x = \frac{1}{4} \pm \frac{\sqrt{3}}{4}}$$