

۱

$$x = 2 \Rightarrow \frac{-b}{2a} = 2 \Rightarrow \frac{-1}{2a-2} = 2 \Rightarrow -1 = 4a-4 \Rightarrow a = \frac{3}{4}$$

$$a = \frac{3}{4} \Rightarrow -\frac{1}{4}x^2 + x + 2 \Rightarrow -x^2 + 4x + 8 \Rightarrow x^2 - 4x - 12 \Rightarrow (x+2)(x-6) \Rightarrow$$

دو نقطه ۲، -۲ قطع می‌کند که عدد ۶ مثبت است.

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چون در ریشه مثبت و دیگر از دورینه منفی است، پس $m < 0$

چون دورینه داریم پس $\Delta > 0$

$$\Delta \Rightarrow b^2 - 4ac \rightarrow 4a - 4m^2 > 0 \rightarrow (a - 2m)(a + 2m) > 0 \quad m < -\frac{a}{2} \text{ or } m > \frac{a}{2}$$

$m \in (-\frac{a}{2}, \frac{a}{2})$ (۲)

استدلال ۱، ۲ $\Rightarrow$ ۱ $\cap$ ۲ $\Rightarrow (-\frac{a}{2}, 0)$

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$$y = x^2 - 5x + p$$

$$s = -\frac{b}{a} \Rightarrow \frac{5 - \sqrt{a}}{2} + \frac{5 + \sqrt{a}}{2} = \frac{10}{2} = 5$$

$$p = \frac{c}{a} \Rightarrow \left(\frac{5 - \sqrt{a}}{2}\right) \left(\frac{5 + \sqrt{a}}{2}\right) = \frac{9 - a}{2} = \frac{9}{2}$$

$$y = x^2 - 5x + \frac{9}{2} \xrightarrow{\times 2} 2x^2 - 10x + 9 = y$$

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$$2x^2 + \beta^2 = 12 \Rightarrow 2x^2 + 2\beta^2 + x^2 - \beta^2 = 12 \Rightarrow 2(x^2 + \beta^2) + x^2 - \beta^2 = 12$$

$$2(x^2 + \beta^2) + (x + \beta)(x - \beta) = 12 \Rightarrow 2(12 - m - 4) + 4\left(\frac{\sqrt{12}}{12}\right) = 12 \Rightarrow$$

$$4 - 2m + 4\left(\frac{\sqrt{48 - 12m - 4}}{2}\right) = 12 \Rightarrow 4 - 2m + 2\sqrt{48 - 12m} = 12 \Rightarrow$$

$$2\sqrt{48 - 12m} = -4 + 2m \Rightarrow 192 - 48m = 4m^2 - 16m + 16 \Rightarrow m^2 - 11m + 14 = 0 \Rightarrow (m - 4)^2 = 0 \Rightarrow m = 4$$

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$$y = a(x-2)^2 + 9 \quad (0, 5) \Rightarrow 5 = a(-2)^2 + 9 \Rightarrow a = -1$$

$$y = -1(x-2)^2 + 9 \Rightarrow -x^2 + 4x - 4 + 9 \Rightarrow y = -x^2 + 4x + 5$$

$$y = -x^2 + 4x + 5 > 0 \Rightarrow x^2 - 4x - 5 < 0 \Rightarrow (x+1)(x-5) < 0 \quad x \in (-1, 5)$$

$x \in (-1, 5)$

$$\alpha x^2 - \sqrt{x} + 9\beta = 0$$

$$\beta > 0$$

$$\alpha x^2 - \sqrt{x} + 9\beta = 0 \rightarrow \alpha x^2 - \sqrt{x} + 9\beta = 0 \rightarrow \alpha x^2 - 9\alpha = 0 \rightarrow x(\alpha x - 9) = 0 \rightarrow \alpha < 0$$

$$\beta^2 \alpha - \sqrt{\beta} + 9\beta = 0 \rightarrow \beta^2 \alpha + \sqrt{\beta} = 0 \rightarrow \beta(\beta \alpha + \sqrt{\beta}) = 0 \rightarrow \beta \alpha + \sqrt{\beta} = 0 \rightarrow \alpha = -\frac{\sqrt{\beta}}{\beta} = -\frac{1}{\sqrt{\beta}}$$

$$\beta > 0 \rightarrow \alpha < 0 \rightarrow \alpha = -\frac{1}{\sqrt{\beta}} \rightarrow \beta = \frac{1}{\alpha^2} = \frac{1}{\alpha^2}$$

$$\frac{1}{\beta} + \frac{1}{\alpha} \rightarrow \frac{\alpha + \beta}{\alpha \beta} = \frac{\sqrt{x}}{9\beta} \rightarrow -\frac{1}{\sqrt{\beta}} + \frac{1}{\alpha} = \frac{\sqrt{x}}{9\beta} = \frac{\sqrt{x}}{9\alpha^2}$$

$$x^2 + mx - 2m = 0 \rightarrow \alpha^2 = -m\alpha + 2m$$

$$\alpha^2 - m\beta = 1 \Rightarrow -m\alpha - m\beta + 2m = 1 \Rightarrow -m(\alpha + \beta - 2) = 1$$

$$m^2 + 2m - 1 = 0 \Rightarrow (m+1)(m-1) = 0 \rightarrow m = -1 \text{ or } m = 1$$

$$m = -1 \rightarrow x^2 - x + 1 = 0 \rightarrow \Delta < 0 \rightarrow \text{no real roots}$$

$$m = 1 \rightarrow x^2 + x - 1 = 0 \Rightarrow \alpha + \beta = -\frac{b}{a} = -1$$

$$y = mx^2 + \epsilon x + \frac{m}{2} + \sqrt{x}$$

$$-2x^2 + \epsilon x + 4 = y$$

$$-2x^2 + \epsilon x + 4 = y \Rightarrow (x-4)(x+2) = 0$$

$$x \rightarrow \begin{cases} -1 \\ 4 \end{cases} \rightarrow \begin{cases} \epsilon = 1 \\ x = 4 \end{cases}$$

$$\frac{-\Delta}{\epsilon a} = 1 \rightarrow \frac{14 - 2m^2 - \epsilon^2}{-2m} = 1 \rightarrow$$

$$-2m^2 - 2\epsilon^2 + 14 = -2m \rightarrow -2m^2 + 2m + 14 - 2\epsilon^2 = 0$$

$$m^2 - \epsilon m - \epsilon^2 = 0 \rightarrow (m-1)(m+\epsilon) = 0$$

$$m = \begin{cases} -1 \\ 1 \end{cases} \rightarrow \begin{cases} -1 \\ 1 \end{cases}$$

$$y = a(x-1)^2 + 4 \xrightarrow{(0, \epsilon)} \epsilon = a(-1)^2 + 4 \Rightarrow \epsilon = 4a + 4 \Rightarrow a = -\frac{1}{4}$$

$$y = -\frac{1}{4}(x-1)^2 + 4 \Rightarrow -\frac{1}{4}x^2 + \frac{1}{2}x - \frac{1}{4} + 4 \Rightarrow -\frac{1}{4}x^2 + \frac{1}{2}x + \frac{15}{4}$$

$$-x^2 + 2x + 15 = f(x) \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha \beta} \Rightarrow \frac{\epsilon}{-1} = \frac{-1}{4}$$

$$\xrightarrow{\epsilon} \epsilon - (\epsilon a + \epsilon)^2 + (\epsilon a^2 + \epsilon a + \epsilon) = 0 \Rightarrow \epsilon - 14a - 1 + \epsilon a^2 + \epsilon a + \epsilon = 0$$

$$\epsilon a^2 - \epsilon a - 1 = 0 \Rightarrow \alpha^2 - \epsilon \alpha - 1 = 0 \Rightarrow (a+1)(a-1) = 0 \Rightarrow a = 1 \text{ or } a = -1$$

$$a = 1 \rightarrow x^2 - 14x + 15 \Rightarrow (x-1)(x-15) = 0 \rightarrow x = 1 \text{ or } x = 15$$

$$a = -1 \rightarrow x^2 - \frac{1}{2}x + \frac{15}{2} \Rightarrow \epsilon x^2 - 14x + \epsilon \Rightarrow x^2 - 14x + 15 \Rightarrow (x-1)(x-15)$$

$$x \in \left\{ \frac{1}{2}, 1, 15 \right\}$$