

$$y = (a-1)x^2 + x + 3$$

$$\text{حالا } a = -\frac{b}{2a} = -\frac{1}{2(a-1)} = -1 \rightarrow a = \frac{2}{3}$$

$$y = \left(\frac{2}{3} - 1\right)x^2 + x + 3 = -\frac{1}{3}x^2 + x + 3 \xrightarrow{y=0} -\frac{1}{3}x^2 + x + 3 = 0 \rightarrow x^2 - 3x - 12 = 0$$

$$\rightarrow x = \frac{3 \pm 1}{2} \rightarrow x_1 = 2, x_2 = -3 \rightarrow \boxed{x_1 = 2}$$

$$f(x) = mx^2 - dx + m$$

$$\Delta = (-d)^2 - 4m^2 = d^2 - 4m^2 \xrightarrow{\Delta \leq 0} d^2 - 4m^2 \leq 0 \rightarrow 4m^2 \leq d^2 \rightarrow |m| \leq \frac{d}{2} \xrightarrow{m < 0} -\frac{d}{2} < m < 0$$

$$\rightarrow \boxed{-\frac{d}{2} < m < 0} \rightarrow \begin{array}{c} -\frac{d}{2} \quad +\frac{d}{2} \\ | \quad | \\ - \quad + \end{array}$$

$$x_1 = \frac{3+\sqrt{5}}{2}, x_2 = \frac{3-\sqrt{5}}{2} \rightarrow x^2 - (x_1+x_2)x + x_1x_2 = 0$$

$$\rightarrow x_1+x_2 = 3, x_1x_2 = \frac{1}{4} \rightarrow x^2 - 3x + \frac{1}{4} = 0 \xrightarrow{\times 4} \boxed{4x^2 - 12x + 1 = 0}$$

$$2x^2 - 11x + m + 2 = 0$$

$$3\alpha^2 + 3\beta^2 = 12 \rightarrow 3\alpha^2(1-\alpha)^2 = 12 \rightarrow 3\alpha^2 - 6\alpha + 3 = 0 \xrightarrow{\div 3} \alpha^2 - 2\alpha + 1 = 0 \rightarrow (\alpha-1)^2 = 0 \rightarrow \boxed{\alpha=1}$$

$$\alpha + \beta = -\frac{b}{a} = \frac{1}{2} = \frac{1}{2} \rightarrow \beta = \frac{1}{2} - \alpha$$

$$\alpha \beta = \frac{c}{a} = \frac{m+2}{2} \rightarrow \alpha\beta = 1 \times \frac{1}{2} = \frac{1}{2} = \frac{m+2}{2} \rightarrow m+2 = 1 \rightarrow \boxed{m=-1}$$

$$\text{حالا } \rightarrow (2, 9) \rightarrow (h, k)$$

$$d_1^2 = x_1^2 = 4$$

$$y = a(x-h)^2 + k \rightarrow y = a(x-2)^2 + 9 \rightarrow -(x-2)^2 + 9 \rightarrow x-2 = \pm 3 \rightarrow x_1 = -1, x_2 = 5$$

$$a = d_1^2 \rightarrow a(5-2)^2 + 9 = 0 \rightarrow 9a + 9 = 0 \rightarrow \boxed{a = -1}$$

$$\rightarrow \boxed{-1 < x < 5} \subseteq (-1, 5)$$

$$ax^2 - vx + 9B = 0$$

$$\frac{1}{a} + \frac{1}{B} = \frac{1}{-r} + \frac{v}{r} = \boxed{\frac{v}{r}}$$

$$x_1 + x_2 = -\frac{b}{a} \sim \frac{v}{a}$$

$$x_1 x_2 = \frac{c}{a} \sim \frac{9B}{a} \rightarrow aB = \frac{9B}{a} = a^2 = 9 \rightarrow a = \pm 3 \rightarrow \begin{cases} a = -3 \\ B = \frac{v}{r} \end{cases}$$

$$x^2 + mx - 5m = 0$$

$$a^2 - mB = 1$$

$$a + B = -m \rightarrow m = -(a + B) \xrightarrow{\text{عوضه}} a^2 - (-(a + B))B = 1 \rightarrow a^2 + B(a + B) = 1 \rightarrow a^2 + aB + B^2 = 1$$

$$aB = -5m$$

$$\rightarrow (a + B)^2 = a^2 + 2aB + B^2 \sim (a + B)^2 - aB = 1 \rightarrow (-m)^2 - (-5m) = 1 \rightarrow m^2 + 5m - 1 = 0$$

$$\hookrightarrow m = \frac{-5 \pm \sqrt{25 + 4}}{2} \rightarrow m_1 = 2 \rightarrow a + B = -2 \rightarrow \begin{cases} a = -3 \\ B = \frac{v}{r} \end{cases}$$

$$f(x) = mx^2 + rx + \frac{m}{r} + v$$

$$\text{مثلاً } a = -\frac{b}{ra} \rightarrow \frac{-r}{rm} = \frac{-r}{m} = k \rightarrow f(k) = 1 \rightarrow f\left(-\frac{r}{m}\right) = m\left(\frac{r}{m^2}\right) + r\left(-\frac{r}{m}\right) + \frac{m}{r} + v$$

$$\rightarrow \frac{r}{m} - \frac{r}{m} + \frac{m}{r} + v \rightarrow f\left(-\frac{r}{m}\right) = -\frac{r}{m} + \frac{m}{r} + v = 1 \rightarrow -\frac{r}{m} + \frac{m}{r} = 1 \rightarrow -1 + m^2 = rm \rightarrow m^2 - rm - 1 = 0$$

$$m = \frac{r \pm \sqrt{r^2 + 4r}}{2} = \frac{r \pm 4}{2} \rightarrow \begin{cases} m = 2 \\ m = -1 \end{cases} \rightarrow \begin{cases} a = -3 \\ B = \frac{v}{r} \end{cases}$$

$$f(x) = a(x - r)^2 + y \xrightarrow{(0, 4)} 4 = a(0 - r)^2 + y = 4 = 4a + y \rightarrow a = -\frac{1}{r}$$

$$f(x) = -\frac{1}{r}(x - r)^2 + y \xrightarrow{f(x) = 0} -\frac{1}{r}(x - r)^2 + y = 0 \rightarrow (x - r)^2 = ry \rightarrow x - r = \pm \sqrt{ry} \rightarrow \begin{cases} x_1 = r - \sqrt{ry} \\ x_2 = r + \sqrt{ry} \end{cases}$$

$$\rightarrow \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1 x_2} = \frac{r + \sqrt{ry} + r - \sqrt{ry}}{r - ry} = \frac{r}{-1} = \boxed{-\frac{1}{r}}$$

$$x^2 - (fa + f)x + (fa^2 + fa + r) = 0$$

$$\rightarrow x_1 + x_2 = fa + f \xrightarrow{x_1 = r} r + x_2 = fa + f \rightarrow x_2 = fa + f - r$$

$$x_1 x_2 = fa^2 + fa + r$$

$$r(fa + f) = fa^2 + fa + r \rightarrow 1a + f = fa^2 + fa + r \rightarrow fa^2 + fa + r - 1a - f = 0$$

$$\rightarrow fa^2 - ra - 1 = 0 \rightarrow \frac{r \pm \sqrt{r^2 + 4r}}{f} \rightarrow \begin{cases} a_1 = 1 \sim x_2 = 4 \\ a_2 = -\frac{1}{f} \sim x_2 = \frac{r}{f} \end{cases}$$