

$$\Delta = \frac{-b}{2a} = v \Rightarrow \frac{(b^2 - 4(-1)(v))}{4(-1)} = v \Rightarrow b^2 + 4v = 4v \Rightarrow b^2 = 14 \Rightarrow b = \pm \sqrt{14}$$

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الف) $f(x) = x^2 + 2x^2 + 3 \xrightarrow{x^2=t} t^2 + 2t + 3 = 0 \Rightarrow \Delta = (2)^2 - 4(1)(3) = -8 < 0$
 (ریشه ندارد)

ب) $f(x) = (4 - x^2)^2 - 2(4 - x^2) - 14 \xrightarrow{4 - x^2=t} t^2 - 2t - 14 = 0 \Rightarrow (t - 4)(t + 3) = 0$
 $\Rightarrow t = 4, -3 \xrightarrow{t=4-x^2} 4 - x^2 = 4 \rightarrow x^2 = 0 \xrightarrow{x^2=0} x = 0$
 $\rightarrow 4 - x^2 = -3 \rightarrow x^2 = 7 \Rightarrow x = \pm \sqrt{7}$

$4x^2 - 14x + m = 0 \rightarrow$ ریشه ها: $\alpha, \alpha + 2$

جمع ریشه ها: $S = 5 = \alpha + \alpha + 2 = 2\alpha + 2 \xrightarrow{S = \frac{-b}{a}} 2\alpha + 2 = \frac{-(-14)}{4} \rightarrow 2\alpha = 2 \rightarrow \alpha = 1$
 ریشه ها: $1, 3$

ضرب ریشه ها: $P = \frac{c}{a} = 1 \times 3 \rightarrow \frac{m}{4} = 3 \rightarrow m = 12$

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$3x^2 - 4x + m - 1 = 0 \rightarrow$ ریشه ها: α, β

$2\alpha + \beta = 4 \rightarrow \beta = 4 - 2\alpha$

جمع ریشه ها: $S = \alpha + \frac{4 - 2\alpha}{1} = 3\alpha - 4 \xrightarrow{S = \frac{-b}{a}} 3\alpha - 4 = \frac{-(-4)}{3} \rightarrow \alpha = 2$

$\beta = 4 - 2\alpha = 4 - 2(2) = 0 \rightarrow \beta = 0$

ضرب ریشه ها: $P = 0 \times 2 = 0 \xrightarrow{P = \frac{c}{a}} \frac{m-1}{3} = 0 \rightarrow m = 1$

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$-mx^2 + 4x + m^2 - 2 = 0 \rightarrow$ ریشه ها: $\alpha, \frac{1}{\alpha}$

ضرب ریشه ها: $P = \frac{1}{\alpha} \times \alpha = 1 \xrightarrow{P = \frac{c}{a}} 1 = \frac{m^2 - 2}{-m} \rightarrow m^2 - 2 = -m \rightarrow m^2 + m - 2 = 0$
 $\rightarrow m = 1, -2$

$m = 1 \rightarrow -x^2 + 4x - 1 = 0 \rightarrow \Delta = 14 - 4(1)(-1) > 0 \rightarrow m = 1$

$m = -2 \rightarrow 2x^2 + 4x + 2 = 0 \rightarrow \Delta = 14 - 4(2)(2) = 0 \rightarrow$ ریشه ها متباین $\Rightarrow m \neq -2$
 (نیست)

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$$\alpha \beta^2 = 4 \rightarrow \alpha = \frac{4}{\beta^2} \rightarrow \text{ضرب در } \beta \rightarrow \frac{4}{\beta^2} \times \beta = \frac{4}{\beta} \xrightarrow{P=\frac{c}{a}} \frac{4}{\beta} = 3 \rightarrow \beta = \frac{4}{3}$$

$$\alpha \left(\frac{4}{3}\right)^2 = 4 \Rightarrow \alpha = \frac{9}{4}$$

$$\text{ریشه } (S) = \frac{9}{4} + \frac{4}{3} = \frac{49}{12} \xrightarrow{S=\frac{-b}{a}} \frac{49}{12} = \frac{-(-m)}{1} \rightarrow m = \frac{49}{12}$$

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$$x^2 - 4x + m = 0 \rightarrow \text{ریشه ها: } \alpha \text{ و } 3\alpha \xrightarrow{1} 1 \text{ و } 3$$

$$\text{جمع ریشه } (S) = \alpha + 3\alpha = 4\alpha \xrightarrow{S=\frac{-b}{a}} 4\alpha = \frac{-(-4)}{1} \rightarrow \alpha = 1 \quad ①$$

$$\text{ضرب ریشه } (P) = 1 \times 3 = 3 \xrightarrow{P=\frac{c}{a}} \frac{m}{1} = 3 \rightarrow m = 3$$

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$$x^2 - 7x + 2 = 0 \rightarrow \text{ریشه ها: } \alpha \text{ و } \beta$$

$$\text{ضرب } P = \frac{c}{a} = 2 = \alpha\beta \rightarrow \beta = \frac{2}{\alpha} \xrightarrow{1} \beta^3 = \frac{1}{\alpha^3} \quad ①$$

$$\alpha^3 + \frac{1}{\alpha^3} \xrightarrow{1} \alpha^3 + \beta^3 = 5^3 - 3(P)(S) \xrightarrow{S=\frac{-b}{a}, P=\frac{c}{a}} \left(\frac{-(-7)}{1}\right)^3 - 3\left(\frac{2}{1}\right)\left(\frac{-7}{1}\right)$$

$$= 343 - 42 = 301$$

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$$h(t) = -1 \cdot t^2 + \Delta \cdot t \Rightarrow \text{زمان برخورد با زمین } h=0 \rightarrow -1 \cdot t^2 + \Delta \cdot t = 0 \rightarrow 1 \cdot t(-t + \Delta) = 0$$

$\rightarrow t=0$ نقطه برخورد با زمین
نقطه شروع پرتاب

$$h_{\max} = y_s = \frac{-\Delta}{4a} = \frac{-((\Delta)^2 - 4(1)(0))}{4(-1)} = \frac{2\Delta \cdot \Delta}{4} = \Delta^2/2$$

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$$(n-1)^2 = 2n+1 \rightarrow n^2 - 2n + 1 = 2n + 1 \rightarrow n^2 = 4n$$

$$n^2 - 4n = 0 \rightarrow n(n-4) = 0 \rightarrow n=0 \text{ و } 4$$

$$\text{نقطه برخورد} = (0,0) \text{ و } (4,9)$$

۲ محل برخورد

$$\Rightarrow n^2 + 2n = |n+2|$$

$$n \geq -2 \quad ①$$

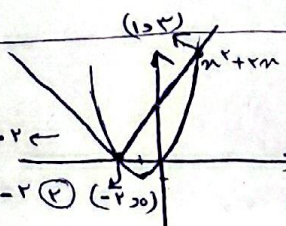
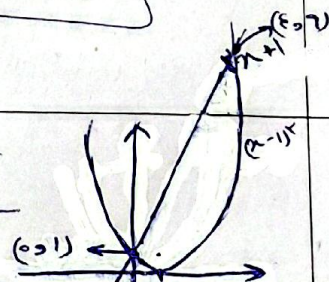
$$n^2 + 2n = n+2 \rightarrow n^2 + n - 2 = 0 \rightarrow n = 1 \text{ و } -2 \quad ②$$

$$① \cap ② \rightarrow n = 1 \text{ و } -2$$

$$n \leq -2 \quad ③$$

$$n^2 + 2n = -n-2 \rightarrow n^2 + 3n + 2 = 0 \rightarrow n = -1 \text{ و } -2 \quad ④$$

$$③ \cap ④ \rightarrow n = -2$$



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