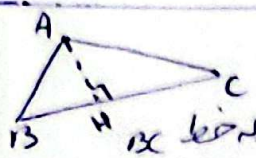


$$m = \frac{1+4}{4-1} = \frac{5}{3} = -V \Rightarrow m' = \frac{1}{V} \quad \text{①}$$

$$x = \frac{2+1}{4} = 3 \quad y = \frac{1-4}{4} = 1 \Rightarrow Z(3, 1) \rightarrow \text{نقطه تقاطع}$$

$$\textcircled{2} \Rightarrow y - y_1 = m(x - x_1) \Rightarrow y - 1 = \frac{1}{V}(x - 3) \Rightarrow y = \frac{x}{V} - \frac{3}{V} + 1$$

$$y = \frac{x}{V} + \frac{4}{V}$$



$$BC = \sqrt{(-1+4)^2 + (2-1)^2} = \sqrt{10} = \omega$$

②

$$\text{مساحت } \Delta ABC = \frac{1}{2} \times BC \times AH = \frac{1}{2} \times \omega \times h \Rightarrow y = V = \frac{4}{V}(x+1) \Rightarrow y = \frac{4x}{V} - \frac{4}{V} + V$$

$$y = \frac{4x}{V} + \frac{4}{V} \Rightarrow 3y = -4x + 4 \Rightarrow 3y + 4x - 4 = 0$$

$$AH = \frac{|4x + 3y - 4|}{\sqrt{4^2 + 3^2}} = \frac{1}{\omega} \Rightarrow d_{BC} - d_{AH} = 4$$

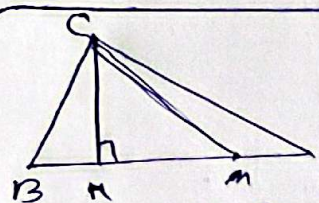
$$\text{نقطه موازی} \Rightarrow \begin{cases} y - \frac{1}{V}x = 3 \\ y - \frac{1}{V}x = \frac{1}{V} \end{cases} \Rightarrow \frac{|c - c'|}{\sqrt{a^2 + b^2}} = \frac{|3 - \frac{1}{V}|}{\sqrt{1 + \frac{1}{V^2}}} = \frac{3}{\sqrt{\frac{V^2 + 1}{V^2}}} = \frac{3V}{\sqrt{V^2 + 1}} = \frac{4}{\sqrt{10}} \Rightarrow \text{مساحت}$$



$$\text{موازی: } 4y - x = V \Rightarrow 4y = x + V \Rightarrow y = \frac{x}{4} + \frac{V}{4} \Rightarrow m = \frac{1}{4} \Rightarrow$$

$$4y = ax + 1 \Rightarrow y = \frac{a}{4}x + \frac{1}{4} \Rightarrow y = \frac{1}{4}x + \frac{1}{4}$$

$$\text{مساحت: } \frac{3}{\sqrt{10}} \Rightarrow \text{مساحت دایره } \pi r^2 \Rightarrow \pi \left(\frac{3}{\sqrt{10}}\right)^2 = \pi \frac{9}{10} = \frac{9}{10}\pi$$



$$C(-1, -2), B(4, 1), A(2, 3)$$

③

$$m = \frac{4+2}{1-2} = -6, y_m = \frac{4+1}{1-2} = -5 \Rightarrow m = (-6, -5)$$

$$AC = \sqrt{(-1-2)^2 + (-2-3)^2} = \sqrt{34}$$

$$AH = \frac{|-1-2-3-6|}{\sqrt{1+1}} = \frac{12}{\sqrt{2}} = 6\sqrt{2}. CH^2 + HM^2 = CM^2 \Rightarrow 12^2 = \frac{11}{2} = HM^2 \Rightarrow \sqrt{\frac{11}{2}} = HM \Rightarrow \frac{1}{\sqrt{2}} \cdot HM$$

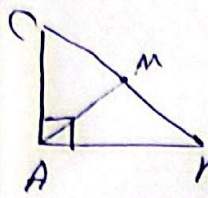
$$\text{③} \Rightarrow \frac{3-1}{4-2} = \frac{1}{2} = -1 \Rightarrow y - 3 = -1(x - 2) \Rightarrow y = -x + 5 \Rightarrow y + x - 5 = 0$$

$$\begin{cases} (m+1)x - (2m-5)y + 1 = 0 \Rightarrow m = \frac{m+1}{-2m+5} \\ (2m-2)x - (5-m)y - 1 = 0 \Rightarrow m = \frac{2m-2}{-5+m} \end{cases} \Rightarrow \frac{2m-2}{-5+m} = \frac{-1}{\frac{m+1}{-2m+5}}$$

$$\frac{2m-2}{-5+m} = \frac{2m-5}{m+1} \Rightarrow (2m-2)(m+1) = (2m-5)(-5+m) \Rightarrow 2m^2 - 12m + 2 = -10m + 25 + m^2 - 10m + 25 \Rightarrow m^2 - 12m + 2 = m^2 - 20m + 50 \Rightarrow 8m = 48 \Rightarrow m = 6$$

$$\Delta = 144 - 4 \cdot 2 = 136 \Rightarrow \sqrt{136} = 2\sqrt{34}$$

$$\begin{cases} AB: x+y=2 \\ AC: y=2m-1 \\ BC: x+y=5 \end{cases}$$



$$\begin{cases} AB: y = -\frac{1}{x}x + 2 \\ AC: y = 2m - 1 \end{cases} \Rightarrow \begin{cases} \text{Midpoint of } AC \text{ is } (\frac{1}{2}, \frac{2m-1}{2}) \\ \text{Line } AM \text{ is } y = 2x - 1 \end{cases}$$

$$-m + 2 = -\frac{1}{2} + \frac{2m-1}{2} \Rightarrow \frac{1}{2}m = \frac{1}{2} \Rightarrow m = 1 \Rightarrow y = -x + 2 \Rightarrow B(1, 1)$$

$$AM = \frac{BC}{2} = \frac{\sqrt{(1-1)^2 + (1-1)^2}}{2} = \frac{\sqrt{0}}{2} = 0 \Rightarrow C(1, 1)$$

$$d_{AH} = \sqrt{1+1} = \sqrt{2} \Rightarrow \frac{\omega\sqrt{2}}{\frac{1}{\sqrt{2}}} = \frac{\omega}{\frac{1}{\sqrt{2}}}$$

$$m = \frac{1}{x} \Rightarrow y = -\frac{1}{x}x + b \Rightarrow -\frac{1}{x}x - y + b = 0 \Rightarrow \sqrt{2} = \frac{|b|}{\sqrt{\frac{1}{x^2} + 1}}$$

$$\sqrt{2} = \frac{|b|}{\sqrt{\frac{1}{x^2} + 1}} \Rightarrow |b| = \omega \Rightarrow \frac{1}{x}x + b = 0 \Rightarrow x = -b \Rightarrow A(-b, 0)$$

$$y = (1-m)x + 2 \quad y = \frac{r}{m}x + 2 \quad y_1 = y_2 \Rightarrow (1-m)x + 2 = \frac{r}{m}x + 2 \Rightarrow x = 0, y = 2 \Rightarrow A(0, 2)$$

$$\begin{cases} (1-m)x + 2 = 0 \Rightarrow x_1 = \frac{r}{m-1} \\ \frac{r}{m}x + 2 = 0 \Rightarrow x_2 = -\frac{m}{r} \end{cases} \Rightarrow (x_1, 0) \cdot (x_2, 0) \cdot (0, 2) \Rightarrow h_{\Delta} = 2$$

$$S_{\Delta} = \frac{1}{2} \cdot 2 \cdot |x_1 - x_2| = |x_1 - x_2| \Rightarrow \left| \frac{r}{m-1} + \frac{m}{r} \right| = \frac{\omega}{r} \Rightarrow$$

$$\begin{cases} \frac{r}{m-1} + \frac{m}{r} = \frac{\omega}{r} \Rightarrow m = 3 \\ \frac{r}{m-1} + \frac{m}{r} = -\frac{\omega}{r} \Rightarrow m = -1 \end{cases} \Rightarrow P(-1, \sqrt{2}) \cdot (-1, \sqrt{2}) = -3$$

$$\begin{cases} m_M = \frac{m_B + m_B'}{2} \Rightarrow 2 = \frac{6 + m_B'}{2} \Rightarrow m_B' = 2 \\ y_M = \frac{y_B + y_B'}{2} \Rightarrow -4 = \frac{-3 + y_B'}{2} \Rightarrow y_B' = -5 \end{cases} \Rightarrow d' = \frac{m}{r} - 3$$

$$\begin{cases} A(1, 2) \\ m_{AA'} = -1 \Rightarrow y - 2 = -1(x - 1) \Rightarrow y = -x + 3 \end{cases} \Rightarrow y = 1, x = 2 \Rightarrow B(2, 1)$$

$$x' = \omega, y' = 1 \Rightarrow A'(\omega, 1) \quad 2b - a = 2 \times 1 - \omega = 2 - \omega$$