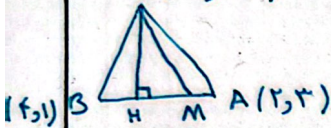


$$A(4, -2) \rightarrow m = \frac{1 - (-2)}{2 - 4} = \frac{1}{-2} = -\frac{1}{2} \xrightarrow{\text{مردم}} \left(\frac{1}{2}\right)$$

$$B(2, 1)$$

$$\text{انتهای مبنی} \rightarrow \left(\frac{4+2}{2}, \frac{-2+1}{2}\right) \rightarrow (3, 0) \rightarrow y-0 = \frac{1}{2}(x-3) \rightarrow y = \frac{1}{2}x + \frac{3}{2}$$

$$C(-1, -3)$$



$$M = \left(\frac{4+2}{2}, \frac{1+3}{2}\right) \rightarrow (3, 2)$$

$$m_{AB} = \frac{1-3}{2-4} = \frac{-2}{-2} = 1 \rightarrow y-3 = 1(x-2) \rightarrow y = x+1$$

$$m_{CH} = 1 \rightarrow y+3 = 1(x-1) \rightarrow y = x-4$$

$$\Rightarrow \begin{cases} y = -x+5 \\ y = x-2 \end{cases} \rightarrow x-2 = -x+5 \rightarrow 2x = 7 \rightarrow x = \frac{7}{2} = 3.5$$

$$MH = \sqrt{(3.5-2)^2 + (1.5-2)^2} = \sqrt{0.5^2 + (-0.5)^2} = \sqrt{0.5} = \frac{1}{\sqrt{2}}$$

$$2y-x=5 \rightarrow y = \frac{1}{2}x + \frac{5}{2}$$

$$4y=5x+2 \rightarrow y = \frac{5}{4}x + \frac{1}{2}$$

$$d = \left| \frac{5}{4} - \frac{1}{2} \right| = \frac{3}{4} = \frac{3}{\sqrt{16}} = \frac{3}{4} \text{ (مقدار)} \xrightarrow{\text{عبارت}} r = \frac{3}{4}$$

$$\text{مساحت} = \pi r^2 = \pi \left(\frac{3}{4}\right)^2 = \pi \cdot \frac{9}{16} = \frac{9\pi}{16}$$

$$BC = \sqrt{(-1+4)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$$

$$\text{مساحت} = \frac{1}{2} (4(3-1) + (-1)(3-1) + (-1)(1-3)) = \frac{1}{2} (4(2) - 1(2) - 1(-2)) = \frac{1}{2} (8 - 2 + 2) = \frac{1}{2} (8) = 4$$

$$\rightarrow BC \cdot AH = 4 \cdot 2 = 8$$

$$\text{②} \rightarrow m_{BC} = \frac{3-1}{-1-4} = \frac{2}{-5} = -\frac{2}{5} \rightarrow y-1 = -\frac{2}{5}(x+1) \rightarrow y = -\frac{2}{5}x + \frac{3}{5}$$

$$(4m+5)x + (d-m)y - 5 = 0 \rightarrow y = -\frac{4m+5}{d-m}x + \frac{5}{d-m} \rightarrow m_1 = -\frac{4m+5}{d-m}$$

$$(m+1)x - (2m-4)y + 1 = 0 \rightarrow y = -\frac{m+1}{2m-4}x - \frac{1}{2m-4} \rightarrow m_2 = -\frac{m+1}{2m-4}$$

$$m_1 \cdot m_2 = -1 \rightarrow \left(-\frac{4m+5}{d-m}\right) \left(-\frac{m+1}{2m-4}\right) = -1$$

$$\frac{(4m+5)(m+1)}{(d-m)(2m-4)} = -1 \rightarrow (4m+5)(m+1) = -(d-m)(2m-4)$$

$$4m^2 + 9m + 5 = -2m^2 + 4m + 2d - 4m \rightarrow 6m^2 + 5m + 5 - 2d = 0 \rightarrow \Delta < 0 \rightarrow \text{مستقیم}$$

$$\begin{cases} AB = x+y = r \\ AC = y = rx-1 \\ BC = x+y = r \end{cases} \rightarrow \begin{cases} x+y = r \\ y = rx-1 \end{cases} \rightarrow x+r(2x-1) = r \rightarrow x+2x-r = r \rightarrow 3x = 2 \rightarrow x = \frac{2}{3}$$

$$y = r - x = r - \frac{2}{3} = \frac{r-2}{3} \rightarrow A = (\frac{2}{3}, \frac{r-2}{3})$$

$$AB, BC = (r-y) + y = r \rightarrow r+y = r \rightarrow y = 0$$

$$x = r+1 = 2 \rightarrow B = (2, -1)$$

$$\rightarrow AC, BC \rightarrow r-x = rx-1 \rightarrow rx = r \rightarrow x = \frac{r}{r-1} \rightarrow y = \frac{r}{r-1}$$

$$C = (\frac{r}{r-1}, \frac{r}{r-1})$$

$$AM = \sqrt{(\frac{2}{3}-1)^2 + (\frac{r-2}{3}-1)^2} = \sqrt{\frac{1}{9} + \frac{(r-5)^2}{9}} = \frac{\sqrt{1+(r-5)^2}}{3}$$

$$AH = d = \frac{|Ax+By+C|}{\sqrt{A^2+B^2}} = \frac{|1x-1y+1|}{\sqrt{1+1}} = \frac{|x-y+1|}{\sqrt{2}}$$

$$\frac{AM}{AH} = \frac{\frac{\sqrt{1+(r-5)^2}}{3}}{\frac{|x-y+1|}{\sqrt{2}}} = \frac{\sqrt{2} \sqrt{1+(r-5)^2}}{3|x-y+1|}$$

$$m = -\frac{1}{r}$$

$$y = -\frac{1}{r}x + c \rightarrow y = -\frac{1}{r}x + d \xrightarrow{y=0} x = 1 \rightarrow (1, 0)$$

$$y = -\frac{1}{r}x + c \rightarrow y = -\frac{1}{r}x - d \xrightarrow{y=0} x = -1 \rightarrow (-1, 0)$$

$$d = \frac{|c|}{\sqrt{1+m^2}} = \frac{|c|}{\sqrt{1+\frac{1}{r^2}}} = \frac{|c|}{\sqrt{\frac{r^2+1}{r^2}}} = \frac{|c|}{\frac{\sqrt{r^2+1}}{r}} = \frac{r|c|}{\sqrt{r^2+1}}$$

$$|c| = d \sqrt{r^2+1}$$

$$my - rx = rm \rightarrow y = \frac{rx}{m} + r \xrightarrow{y=0} 0 = \frac{rx}{m} + r \rightarrow x = -\frac{m}{r} \rightarrow x_1 = (-\frac{m}{r}, 0)$$

$$y + mx = x + r \rightarrow y = x(1-m) + r \xrightarrow{y=0} 0 = x(1-m) + r \rightarrow x = -\frac{r}{1-m} \rightarrow x_2 = (-\frac{r}{1-m}, 0)$$

$$S = \frac{1}{r} \times r |x_2 - x_1| = |x_2 - x_1| = |-\frac{r}{1-m} - (-\frac{m}{r})| = |-\frac{r}{1-m} + \frac{m}{r}|$$

$$\rightarrow dm - d = -m^2 + m - r \rightarrow m^2 + rm - 1 = 0$$

$$-dm + d = -m^2 + m - r \rightarrow m^2 - 4m + 4 = 0 \rightarrow (m-2)^2 = 0 \rightarrow m = 2$$

$$S = 2x - 1 = -1$$

$$y = rx - r \rightarrow m = (r, -r)$$

$$\rightarrow x = rx - x, y = rx - y$$

$$x = r-x, y = -r-y \rightarrow y = -r - (rx - r) = -r - rx + r = -rx \rightarrow x = \frac{-y}{r}$$

$$-r(r-x) - 1 = -1 + rx - 1 = rx - 1$$

$$A(1, 2)$$

$$y = x + a \rightarrow A' = xA' = rxB - xA = r(2) - 1 = 1 \rightarrow A' = (1, 1)$$

$$yA' = ryB - yA = 1 - 2 = -1 \rightarrow A' = (-1, 1)$$

$$rb - a = ? \rightarrow r(2) - 1 = 1$$

$$y - r = -x + 1 \rightarrow y = -x + r$$

$$\rightarrow -x + r = x + a \rightarrow rx = -r \rightarrow x = -1, y = r$$