

$A(4, -4), B(7, 1)$

$m = \frac{-4-1}{4-7} = \frac{-5}{-3} = \frac{5}{3}$

$m' = \frac{1}{5}$

$M \mid \begin{cases} \frac{x+y}{2} = 3 \\ \frac{-4+1}{-3} = 1 \end{cases}$

$y-1 = \frac{1}{5}(x-7) \Rightarrow y = \frac{1}{5}x + \frac{2}{5}$

$A(7, 5), B(-4, 5), C(-1, 7)$

$m_{BC} = \frac{5-7}{-4-(-1)} = \frac{-2}{-3} = \frac{2}{3}$ $y-7 = -\frac{2}{3}(x+1) \Rightarrow y = -\frac{2}{3}x + \frac{20}{3} \Rightarrow -2x - 3y + 20 = 0$

$AH = \frac{|-2(7) - 3(5) + 20|}{\sqrt{4+9}} = \frac{10}{5} = 2$ $BC = \sqrt{(-4-(-1))^2 + (5-7)^2} = \sqrt{9+4} = \sqrt{13} = \sqrt{13}$

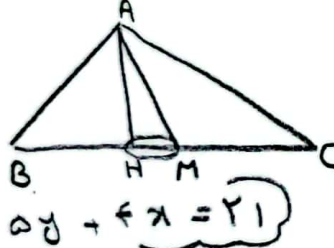
$5-7 = -2$

$2y-x = 5, 4y = x+7$

$\begin{cases} x-2y+5=0 \\ 2x-4y+7=0 \end{cases} \rightarrow \begin{cases} 2x-4y+10=0 \\ 2x-4y+7=0 \end{cases} \rightarrow d = \frac{|10-7|}{\sqrt{4+16}} = \frac{3}{\sqrt{20}} = \frac{3\sqrt{5}}{10} = \frac{3}{2\sqrt{5}}$

$EW = \frac{1}{2} \times \frac{3}{\sqrt{5}} = \frac{3}{2\sqrt{5}} \rightarrow S = (\frac{3}{\sqrt{5}})^2 \pi = \frac{9}{5} \pi$

$A(7, 5), B(4, 1), C(-1, -3)$ $M(3, 2)$ CH $M(3, 2)$



$m_{BC} = \frac{-3-1}{-1-4} = \frac{-4}{-5} = \frac{4}{5}$ $y-1 = \frac{4}{5}(x-4) \Rightarrow y = \frac{4}{5}x - \frac{11}{5}$

$m_{AH} = -\frac{5}{4} \Rightarrow y-5 = -\frac{5}{4}(x-7) \Rightarrow 4y-20 = -5x+35 \Rightarrow 5x+4y-55=0$

$m_{AB} = -1$ $m_{CH} = 1$

$CH: y = x-4$ $m_H = \frac{|4-11+1|}{\sqrt{1+16}} = \frac{6}{\sqrt{17}}$

$4y+5x=22$

$\begin{cases} -20y-14x=-14 \\ 4y+5x=22 \end{cases} \rightarrow \begin{cases} -20y-14x=-14 \\ 8y+10x=44 \end{cases}$

$9x=24 \Rightarrow x=\frac{8}{3}, y=\frac{11}{3} \rightarrow H(\frac{8}{3}, \frac{11}{3})$

$MH = \sqrt{(\frac{8}{3}-\frac{7}{2})^2 + (\frac{11}{3}-2)^2}$

① $(5m-2)x + (5-m)y - 5 = 0$ $m_1 = \frac{-(5m-2)}{5-m}$ $\rightarrow \frac{-(5m-2)}{5-m} \times \frac{m+1}{5m-2} = -1$

② $(m+1)x - (5m-2)y + 1 = 0$ $m_2 = \frac{-(m+1)}{5m-2}$

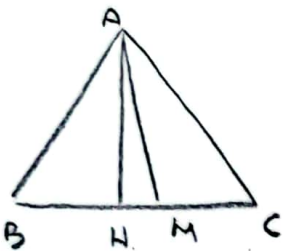
$\rightarrow (5m-2)(m+1) = (5-m)(5m-2) \Rightarrow 5m^2 + 3m - 2 = 25m - 5m^2 - 25 + 10m - 2 \Rightarrow 10m^2 - 12m + 1 = 0$

$5m^2 + m - 2 = -5m^2 + 12m - 20 \rightarrow 10m^2 - 11m + 18 = 0$

$$\begin{cases} AB: x + 2y = 3 \\ AC: y = 2x - 1 \\ BC: x + y = 1 \end{cases} \rightarrow \begin{cases} x + 2y = 3 \\ -2x + y = -1 \end{cases} \rightarrow \begin{cases} 2x + 4y = 6 \\ -2x + y = -1 \end{cases} \rightarrow \begin{cases} x + y = 3 \\ x = 1 \end{cases} \rightarrow A(1, 1)$$

$$\begin{cases} x + 2y = 3 \\ x + y = 1 \end{cases} \rightarrow \begin{cases} x - 1 = 1 \\ x = 0 \end{cases} \rightarrow B(0, -1)$$

$$\begin{cases} -2x + y = -1 \\ x + y = 1 \end{cases} \rightarrow \begin{cases} -x + y = 0 \\ -3x = -2 \end{cases} \rightarrow \begin{cases} y = x \\ x = \frac{2}{3} \end{cases} \rightarrow C(\frac{2}{3}, \frac{2}{3})$$

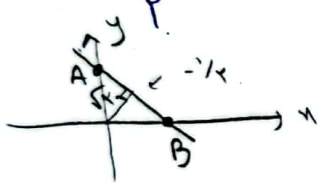


$$M: (\frac{1}{2}, \frac{1}{2}) \quad AM = \sqrt{(1 - \frac{1}{2})^2 + (1 - \frac{1}{2})^2} = \sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{1} = 1$$

$$AH = \frac{|1 + 1 - 1|}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\frac{AM}{AH} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

$$m = \frac{-1}{2}$$



$$\text{Line } AB: y = -\frac{1}{2}x + b \rightarrow \frac{1}{2}x + b - y = 0 \quad (1.5)$$

$$\frac{x^2}{2} + x + 2y - 2b = 0 \rightarrow \frac{0 + 0 - 2b}{\sqrt{1 + 4}} = \frac{2|b|}{\sqrt{5}} = \sqrt{20}$$

$$2|b| = \sqrt{100} = 10 \rightarrow |b| = 5 \rightarrow b = \pm 5 \quad AB = \sqrt{125}$$

$$my - 4x = 2m \quad (1) \quad y + mx = x + 2 \rightarrow y + (m-1)x = 2 \quad (2)$$

ابتدا تقاطع دو خط با محور x ها را بدست می آوریم

$$(1) - 4x = 2m \rightarrow x = -\frac{m}{2} \rightarrow A(-\frac{m}{2}, 0)$$

$$(2) (m-1)x = 2 \rightarrow x = \frac{2}{m-1} \rightarrow B(0, \frac{2}{m-1})$$

تقاطع دو خط را بدست می آوریم تا رأس سوم بدست آید

$$\begin{cases} my - 4x = 2m \\ y + (m-1)x = 2 \end{cases} \rightarrow \begin{cases} m(2 - (m-1)x) - 4x = 2m \\ 2m - m(m-1)x - 4x = 2m \end{cases}$$

$$\begin{cases} y + (m-1)x = 2 \rightarrow y = 2 - (m-1)x \\ x[-m^2 + m - 4] = 0 \end{cases} \rightarrow x = 0 \rightarrow y = 2$$

$$x = 0 \rightarrow m^2 - m + 4 = 0 \rightarrow \Delta = -15 < 0 \rightarrow \text{no real roots} \rightarrow x = 0 \rightarrow y = 2$$

محل تقاطع 2 خط و نقطه سوم است C(0, 2)

$$ارتفاع مثلث = 2 \quad AB = \left| \frac{2}{m-1} - 5\left(\frac{m}{2}\right) \right| = \left| \frac{2}{m-1} + \frac{m}{2} \right|$$

(2)

$$S = \frac{1}{2} AB \times 2 = AB \rightarrow AB = \frac{5}{2} \rightarrow \left| \frac{2}{m-1} + \frac{m}{2} \right| = \frac{5}{2}$$

$$\star \frac{2}{m-1} + \frac{m}{2} = \frac{5}{2} \rightarrow \left(\frac{2}{m-1} \right) = \frac{5}{2} - \frac{m}{2} = \frac{5-m}{2} \rightarrow (5-m)(m-1) = 4 \rightarrow$$

$$m^2 - 4m + 4 = 0 \rightarrow (m-2)^2 = 0 \rightarrow m = 2$$

$$\star \frac{2}{3-1} + \frac{3}{2} = -\frac{5}{2} \rightarrow \frac{2}{m-1} = -\frac{5}{2} - \frac{m}{2} = \frac{-5-m}{2} \rightarrow m^2 + 6m - 1 = 0$$

$$m = -2 \pm \sqrt{5}$$

فاصل ضرب مقادیر ممکن برای m :

$$2 \times \underbrace{(-2 + \sqrt{5})(-2 - \sqrt{5})}_{-1} = 2(-1) = -2$$

$$y = 2x - 3 \quad M(2, -2)$$

(9) اگر x و y قرینه‌های متناسب به M باشند

$$\frac{x+x'}{2} = 2, \quad \frac{y+y'}{2} = -2$$

$$\begin{cases} x+x' = 4 \rightarrow x' = 4-x \\ y+y' = -4 \rightarrow y' = -4-y \end{cases}$$

$$\rightarrow \begin{cases} x = 4-x' \\ y = -4-y' \end{cases}$$

$$\rightarrow (-4-y') = 2(4-x') - 3 \rightarrow -4-y' = 8-2x'-3 \rightarrow$$

$$y' = 2x' - 9 \Rightarrow y = 2x - 9$$

$$y = x + 5 \quad A(1, 2) \quad A'(a, b) \quad 2a - b = ? \quad b - a \quad (10)$$

قرینه نقطه A نسبت به خط در خط عمود بر این خط و در فاصله مساوی از خط در

$$m = 1 \rightarrow m_{\perp} = -1$$

$$خط دیگر عمود بر این خط و در فاصله مساوی از خط در محور$$

$$\begin{cases} y = x + 5 \\ y = -x + 3 \end{cases} \rightarrow -x + 3 = x + 5 \rightarrow x = -1 \rightarrow y = -1 + 5 = 4 \rightarrow A'(-1, 4)$$

$$AA' \text{ بر } M : \left(\frac{1+a}{2}, \frac{2+b}{2} \right) = (-1, 4) \rightarrow \begin{cases} 1+a = -2 \rightarrow a = -3 \\ 2+b = 8 \rightarrow b = 6 \end{cases} \rightarrow A'(-3, 6)$$

$$2a - b = -6 - 6 = -12 \quad 10$$