

$A(4, -4), B(7, 1)$

$m = \frac{-4-1}{4-7} = \frac{-5}{-3} = \frac{5}{3}$

$m' = \frac{1}{5}$

$M \mid \begin{cases} \frac{x+y}{2} = 3 \\ \frac{-4+1}{-3} = 1 \end{cases}$

$y-1 = \frac{1}{5}(x-7) \Rightarrow y = \frac{1}{5}x + \frac{2}{5}$

$A(7, 5), B(-4, 5), C(-1, 3)$

$m_{BC} = \frac{5-3}{-4-(-1)} = \frac{2}{-3} = -\frac{2}{3}$ $y-3 = -\frac{2}{3}(x+1) \Rightarrow y = -\frac{2}{3}x + \frac{8}{3} \Rightarrow -2x - 3y + 8 = 0$

$AH = \frac{|-2(7) - 3(5) + 8|}{\sqrt{4+9}} = \frac{10}{5} = 2$ $BC = \sqrt{(-4-(-1))^2 + (5-3)^2} = \sqrt{9+4} = \sqrt{13} = \sqrt{13}$

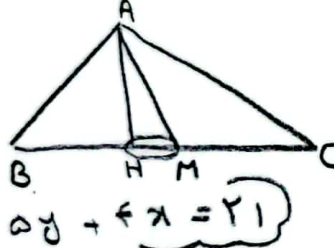
$\Delta = 2 = 2$

$2y - x = 5, 4y = x + 2$

$\begin{cases} x - 2y + 5 = 0 \\ 2x - 4y + 2 = 0 \end{cases} \rightarrow \begin{cases} 2x - 4y + 10 = 0 \\ 2x - 4y + 2 = 0 \end{cases} \rightarrow d = \frac{|10-2|}{\sqrt{4+16}} = \frac{8}{\sqrt{20}} = \frac{4}{\sqrt{5}} = \frac{4}{\sqrt{5}}$

$E = \frac{1}{2} \times \frac{4}{\sqrt{5}} = \frac{2}{\sqrt{5}} \rightarrow S = (\frac{2}{\sqrt{5}}) \times \pi = \frac{2\pi}{\sqrt{5}}$

$A(7, 5), B(-4, 5), C(-1, -3)$ $M = (\frac{7}{2}, -1)$



$m_{BC} = \frac{-4}{-5} = \frac{4}{5} \rightarrow y-1 = \frac{4}{5}(x-4) \Rightarrow y = \frac{4}{5}x - \frac{11}{5}$

$m_{AH} = -\frac{5}{4} \Rightarrow y-5 = -\frac{5}{4}(x-7) \Rightarrow 4y-20 = -5x+35 \Rightarrow 5x+4y-55=0$

$4y + 5x = 55$

$\begin{cases} -5x - 20y = -11 \\ 5x + 4y = 55 \end{cases} \rightarrow \begin{cases} -20y = -11 \\ 5x + 4y = 55 \end{cases}$

$9x = 24 \rightarrow x = \frac{24}{9} = \frac{8}{3}, y = \frac{11}{12} \rightarrow H(\frac{8}{3}, \frac{11}{12})$

$MH = \sqrt{(\frac{8}{3} - \frac{7}{2})^2 + (\frac{11}{12} + 1)^2}$

① $(5m-2)x + (5-m)y - 1 = 0$ $m_1 = \frac{-(5m-2)}{5-m} \rightarrow \frac{-(5m-2)}{5-m} \times \frac{m+1}{5m-2} = -1$

② $(m+1)x - (5m-2)y + 1 = 0$ $m_2 = \frac{-(m+1)}{5m-2}$

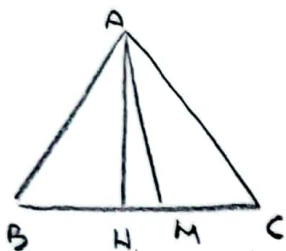
$\rightarrow (5m-2)(m+1) = (5-m)(5m-2) \Rightarrow 5m^2 + 3m - 2 = 25m - 5m^2 - 25 + 10m - 2m^2 + 10m - 2$

$5m^2 + m - 2 = -2m^2 + 14m - 25 \rightarrow 7m^2 - 13m + 23 = 0$

$$\begin{cases} AB: x + 2y = 3 \\ AC: y = 2x - 1 \\ BC: x + y = 1 \end{cases} \rightarrow \begin{cases} x + 2y = 3 \\ -2x + y = -1 \end{cases} \rightarrow \begin{cases} 2x + 4y = 6 \\ -2x + y = -1 \end{cases} \rightarrow \begin{cases} x + y = 1 \\ x + y = 3 \end{cases} \rightarrow \begin{cases} x = 1 \\ y = 1 \end{cases} \rightarrow A(1, 1)$$

$$\begin{cases} x + 2y = 3 \\ x + y = 1 \end{cases} \rightarrow \begin{cases} x + 2y = 3 \\ -x - y = -1 \end{cases} \rightarrow \begin{cases} x + 2y = 3 \\ -x - y = -1 \end{cases} \rightarrow \begin{cases} x = 0 \\ y = -1 \end{cases} \rightarrow B(0, -1)$$

$$\begin{cases} -2x + y = -1 \\ x + y = 1 \end{cases} \rightarrow \begin{cases} -2x + y = -1 \\ -x - y = -1 \end{cases} \rightarrow \begin{cases} -2x + y = -1 \\ -3x = -2 \end{cases} \rightarrow \begin{cases} x = \frac{2}{3} \\ y = \frac{1}{3} \end{cases} \rightarrow C(\frac{2}{3}, \frac{1}{3})$$



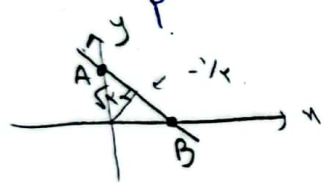
$$M: (\frac{1}{3}, \frac{1}{3}) \quad AM = \sqrt{(1 - \frac{1}{3})^2 + (1 - \frac{1}{3})^2} = \sqrt{(\frac{2}{3})^2 + (\frac{2}{3})^2} = \sqrt{\frac{4}{9} + \frac{4}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

$$\sqrt{\frac{4}{9} + \frac{1}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$$

$$AH = \frac{|1 + 1 - 1|}{\sqrt{1^2 + 1^2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

$$\frac{AM}{AH} = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{\sqrt{2}}} = \frac{2\sqrt{2} \times \sqrt{2}}{3 \times 1} = \frac{4}{3}$$

$$m = \frac{-1}{2}$$



$$\text{Line: } y = -\frac{1}{2}x + b \rightarrow \frac{1}{2}x + b - y = 0$$

$$\frac{x^2}{4} + x + 2y - 2b = 0 \rightarrow \frac{0 + 0 - 2b}{\sqrt{1 + 4}} = \frac{2|b|}{\sqrt{5}} = \sqrt{20}$$

$$2|b| = \sqrt{100} = 10 \rightarrow |b| = 5 \rightarrow b = \pm 5$$

$$my - 4x = 2m \quad (1) \quad y + mx = x + 2 \rightarrow y + (m-1)x = 2 \quad (2)$$

ابتدا تقاطع دو خط با محور x ها را بدست می آوریم

$$(1) -4x = 2m \rightarrow x = -\frac{m}{2} \rightarrow A(-\frac{m}{2}, 0)$$

$$(2) (m-1)x = 2 \rightarrow x = \frac{2}{m-1} \rightarrow B(0, \frac{2}{m-1})$$

تقاطع دو خط را بدست می آوریم تا رأس سوم بدست آید

$$\begin{cases} my - 4x = 2m \\ y + (m-1)x = 2 \end{cases} \rightarrow \begin{cases} my - 4x = 2m \\ y = 2 - (m-1)x \end{cases} \rightarrow m(2 - (m-1)x) - 4x = 2m$$

$$2m - m(m-1)x - 4x = 2m \rightarrow x[-m^2 + m - 4] = 0 \rightarrow x = 0 \rightarrow y = 2$$

$$x = 0 \rightarrow m^2 - m + 4 = 0 \rightarrow \Delta = -15 < 0 \rightarrow \text{No real solution}$$

محل تقاطع 2 خط و نقطه سوم است $C(0, 2)$

$$ارتفاع\ مست = 2 \quad AB = \left| \frac{2}{m-1} - \left(\frac{m}{2} \right) \right| = \left| \frac{2}{m-1} + \frac{m}{2} \right|$$

$$S = \frac{1}{x} AB \times x = AB \rightarrow AB = \frac{5}{2} \rightarrow \left| \frac{2}{m-1} + \frac{m}{2} \right| = \frac{5}{2}$$

$$\star \frac{2}{m-1} + \frac{m}{2} = \frac{5}{2} \rightarrow \left(\frac{2}{m-1} \right) = \frac{5}{2} - \frac{m}{2} = \frac{5-m}{2} \rightarrow (5-m)(m-1) = 4 \rightarrow$$

$$m^2 - 4m + 4 = 0 \rightarrow (m-2)^2 = 0 \rightarrow m = 2$$

$$\star \frac{2}{3-1} + \frac{3}{2} = -\frac{5}{2} \rightarrow \frac{2}{m-1} = -\frac{5}{2} - \frac{m}{2} = \frac{-5-m}{2} \rightarrow m^2 + 6m - 1 = 0$$

$$m = -2 \pm \sqrt{5}$$

فاصل ضرب مقادیر ممکن برای m

$$2 \times \underbrace{(-2 + \sqrt{5})(-2 - \sqrt{5})}_{-1} = 2(-1) = -2$$

$$y = 2x - 3 \quad M(2, -2)$$

اگر x و y قرینه M نسبت به M باشند

$$\frac{x+x'}{2} = 2, \quad \frac{y+y'}{2} = -2$$

$$\begin{cases} x+x' = 4 \rightarrow x' = 4-x \\ y+y' = -4 \rightarrow y' = -4-y \end{cases} \rightarrow \begin{cases} x = 4-x' \\ y = -4-y' \end{cases} \rightarrow (-4-y') = 2(4-x') - 3 \rightarrow$$

$$-4-y' = 8-2x'-3 \rightarrow y' = 2x'-9 \Rightarrow y = 2x-9$$

$$y = x + 5 \quad A(1, 2) \quad A'(a, b) \quad 2a - b = ?$$

قرینه نقطه A نسبت به خط در خط عمود بر این خط و در فاصله مساوی از خط در طرف دیگر قرار دارد

$$m=1 \rightarrow m_{\perp} = -1$$

$$معادله خط عمود $y - 2 = -1(x - 1) \rightarrow y = -x + 1 + 2 \rightarrow y = -x + 3$$$

$$\begin{cases} y = x + 5 \\ y = -x + 3 \end{cases} \rightarrow -x + 3 = x + 5 \rightarrow x = -1 \rightarrow y = -1 + 5 = 4 \rightarrow \text{منشأ خط}$$

$$AA' \text{ ب } M : \left(\frac{1+a}{2}, \frac{2+b}{2} \right) = (-1, 4) \rightarrow \begin{cases} 1+a = -2 \rightarrow a = -3 \\ 2+b = 8 \rightarrow b = 6 \end{cases} \rightarrow A'(-3, 6)$$

$$2a - b = -6 - 6 = -12$$