

$$A(-r, k) \quad B(\varepsilon, m) \quad m = -\frac{1}{r} \quad \text{مستقيم}$$

مسألة 2

ABCD مربع $\Rightarrow 4 \times 4$

$$\frac{m-k}{\varepsilon-(-r)} = -\frac{1}{r} \rightarrow rm - rk = -\varepsilon \rightarrow m - k = -\frac{\varepsilon}{r}$$

$$AB = (4, m-k) = (4, -\frac{\varepsilon}{r})$$

$$|AB| = \sqrt{4^2 + \frac{\varepsilon^2}{r^2}} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$4 \times \sqrt{25} = 20 = \varepsilon$$

$$A(-1, \varepsilon), B(r, 1), C(m, y), D(-1-m, y+r)$$

مسألة 2

$$AB \parallel CD \rightarrow AB = (r-(-1), 1-\varepsilon) \rightarrow (r+1, 1-\varepsilon)$$

$$CD = (-1-m-m, y+r-y) \rightarrow (-1-2m, r)$$

$$\frac{-1-2m}{\varepsilon} = \frac{r}{1-\varepsilon} \rightarrow -1-2m = \frac{r\varepsilon}{1-\varepsilon} \rightarrow 2m = -1 - \frac{r\varepsilon}{1-\varepsilon} \rightarrow m = -\frac{1}{2} - \frac{r\varepsilon}{2(1-\varepsilon)}$$

$$|AB| = \sqrt{(r+1)^2 + (1-\varepsilon)^2} = d$$

$$|CD| = d$$

$$BC = (m-r, y-1) = (-\frac{r}{2}, y-1)$$

$$AB \perp BC \rightarrow \overrightarrow{AB} \cdot \overrightarrow{BC} = 0$$

$$(r+1, 1-\varepsilon) \cdot (-\frac{r}{2}, y-1) = 0$$

$$-\frac{r(r+1)}{2} + (1-\varepsilon)(y-1) = 0$$

$$|BC| = \sqrt{(\frac{r}{2})^2 + (y-1)^2} = \sqrt{\frac{r^2}{4} + (y-1)^2} = \sqrt{\frac{r^2}{4} + \frac{r^2}{4}} = \sqrt{\frac{r^2}{2}} = \frac{r}{\sqrt{2}}$$

$$P = 2(|AB| + |BC|) = 2(d + \frac{r}{\sqrt{2}}) = 2(\frac{r}{\sqrt{2}} + \frac{r}{\sqrt{2}}) = 2\sqrt{2}r$$

$$rmn + (m^2 - 1)y = r$$

m, n

$\frac{y}{\sqrt{r}}, \tan$

$$K = -\frac{rm}{m^2 - 1} = \sqrt{r}$$

$$\rightarrow \sqrt{r}m^2 - \sqrt{r} + rm = 0$$

$$\Delta = \varepsilon - \varepsilon(\sqrt{r})(\sqrt{r}) = 19$$

$$\frac{-r \pm \varepsilon}{2\sqrt{r}} = \frac{-4 \pm \varepsilon}{2\sqrt{r}} = -\sqrt{r}$$

$$|m, -m| = \frac{1}{\sqrt{r}} - (-\sqrt{r}) = \frac{\varepsilon}{\sqrt{r}} = \frac{\varepsilon\sqrt{r}}{r}$$

$$A(1, 9) \quad B(2, 2) \quad C(11, 1)$$

مسئله هندسی

طول ارتفاع

$$m = \frac{11-2}{1-2} = \frac{9}{-1} = -9$$

$$y = mx + b \rightarrow y = -9x - 1$$

$$d_{AH} = \frac{|a \cdot x_0 + b \cdot y_0 + c|}{\sqrt{a^2 + b^2}} = \frac{|1 \cdot 1 - 9 \cdot 9 - 1|}{\sqrt{1 + 81}} = \frac{|-81|}{\sqrt{82}} = \frac{81}{\sqrt{82}} = \frac{81\sqrt{82}}{82}$$

$$AB: y - 2 = m(x - 1)$$

$$2(11-2) - 11m = -1 \rightarrow 18 - 11m = -1 \rightarrow -11m = -19 \rightarrow m = \frac{19}{11}$$

$$AC: 9y - 2x = 11$$

$$BC: 2y - 11x = -19$$

$$m = \frac{19}{11}$$

بH ارتفاع = ?

AC و B

$$y = 11 - 2x \rightarrow B(2, 1)$$

$$d = \frac{|-2 \cdot 2 + 11 \cdot 1 - 11|}{\sqrt{4 + 121}} = \frac{|-10|}{\sqrt{125}} = \frac{10}{5\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$a = \frac{y}{x} = \frac{1}{2} \rightarrow y = \frac{1}{2}x$$

$$m = -\frac{1}{2} \rightarrow y = -\frac{1}{2}x + 1$$

$$d = \frac{1}{\sqrt{5}}$$

$$y - am = 1$$

$$d = \frac{1}{\sqrt{5}}$$

$$ay - m = a - 1$$

(1, 2) مرکز دایره

مسئله هندسی

$$y = am + 1 \rightarrow 2 = a + 1 \rightarrow a = 1 \rightarrow y = m + 1$$

$$y = \frac{1}{a}m + \frac{a-1}{a} \rightarrow y - m = 0 \rightarrow y = m$$

$$\frac{|1-0|}{\sqrt{1^2+1^2}} = \frac{1}{\sqrt{2}}$$

$$d^2 = r^2 + \left(\frac{1}{\sqrt{r}}\right)^2 \rightarrow 2a^2 = r^2 + \frac{1}{r} \rightarrow r^2 = \frac{2a^2}{r} \Rightarrow r = \frac{2a}{\sqrt{r}}$$

$$d = \frac{2a}{\sqrt{r}} \times \frac{1}{\sqrt{r}} = \frac{2a}{r} = \frac{2}{\sqrt{r}}$$

$$A(1, 0)$$

$$m - 3y = 1$$

در فضای سه بعدی

$$m - 3y - 1 = 0 \rightarrow A=1, B=-3, C=-1$$

$$d = \frac{|1 \cdot 1 - 3 \cdot 0 - 1|}{\sqrt{1 + 9 + 1}} = \frac{1}{\sqrt{11}}$$

$$y = \frac{x-1}{3}$$

$$S = \frac{1}{r} \times |BC| \times d$$

↓

$$m > 0, y > 0 \rightarrow m - 1 > 0 \rightarrow m > 1$$

$$|BC| = \sqrt{(m_2 - m_1)^2 + \left(\frac{m_2 - m_1}{3}\right)^2} = \sqrt{(m_2 - m_1)^2 \left(1 + \frac{1}{9}\right)} = \frac{\sqrt{10}}{3} (m_2 - m_1) = \frac{\sqrt{10}}{3} (m - 1)$$

$$S = \frac{1}{r} \times |BC| \rightarrow \frac{1}{r} \times \frac{\sqrt{10}}{3} (m - 1) \times \frac{1}{\sqrt{11}} = \frac{1}{r} (m - 1)$$

$$B(1, 0) \rightarrow y = 0 \rightarrow m - 3 \cdot 0 = 1 \rightarrow m = 1$$

$$\frac{1}{r} - \frac{1}{r} = 0$$

$$S \Rightarrow \Delta m = \frac{(1-1)}{r} = 0$$

مسئله ۱

$$P_1 = (-\frac{1}{r}, a) \quad P_2 = (-\frac{1}{r}, b)$$

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(-\frac{1}{r} + \frac{1}{r})^2 + (b-a)^2} = \sqrt{(\frac{1}{r})^2 + (b-a)^2}$$

$$\sqrt{\frac{1}{r^2} + (b-a)^2}$$

بسط و جمع

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b-a}{-\frac{1}{r}} = -r(b-a)$$

$$-r(b-a) = \sqrt{r} \rightarrow b-a = \frac{\sqrt{r}}{r}$$

$$d = \sqrt{\frac{1}{r^2} + (\frac{\sqrt{r}}{r})^2} = \sqrt{\frac{1}{r^2} + \frac{r}{r^2}} = \sqrt{\frac{1+r}{r^2}} = \frac{\sqrt{1+r}}{r}$$

$$d\sqrt{r} = \frac{1}{r} \sqrt{r} \rightarrow \frac{\sqrt{r}}{r}$$

بسط و جمع

$$(-r, -\varepsilon)$$

نقطه ۱

نقطه ۲

$$r = \sqrt{r^2 + \varepsilon^2} = A$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1}{-r}$$

$$m = -\frac{1}{r} = -\frac{y_1}{r} \rightarrow y + \varepsilon = -\frac{y_1}{r}(r + \varepsilon) \rightarrow y = -\frac{y_1}{r}(r + \varepsilon)$$

$$(r, y) \rightarrow (r \cos \varepsilon, r \sin \varepsilon) \rightarrow \boxed{r = \frac{d}{\sqrt{r}}} \quad \boxed{y = \frac{d}{\sqrt{r}}} \quad \boxed{-\frac{d}{\sqrt{r}}}$$