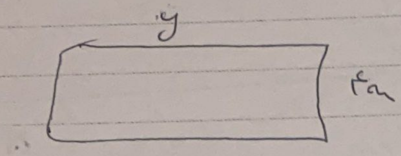
درسا صادقی

20

1ـ

$$\frac{\sqrt{5}+1}{2}=\frac{y}{4x} \qquad 5x\times 4x = 20x^2$$

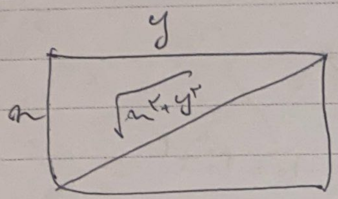


$$y = 2x(\sqrt{5}+1) \longrightarrow 4x\times 2x(\sqrt{5}+1)$$

$$\frac{4x\times 2x(\sqrt{5}+1)}{4x\times 5x} = 0.4\sqrt{5}+0.4$$

2

2ـ



$$\left(\frac{\sqrt{x^2+y^2}}{y}=\frac{\sqrt{5}+1}{2}\right)^2 \longrightarrow \frac{x^2+y^2}{y^2}=\frac{6+2\sqrt{5}}{4}$$

$$2x^2+2y^2=3y^2+y^2\sqrt{5} \longrightarrow 2x^2=y^2+y^2\sqrt{5} \longrightarrow x^2=y^2\left(\frac{1+\sqrt{5}}{2}\right)$$

$$\frac{y^2}{x^2}=\frac{\sqrt{5}-1}{2}$$

2

3ـ

$$3a+\sqrt{2a^2+4a}=2 \longrightarrow \left\{\sqrt{2a^2+4a}=2-3a\right\}^2 \longrightarrow$$

$$2a^2+4a=4+9a^2-12a \longrightarrow 7a^2+4-16a=0$$

$$\Delta = 16 - 4\times 7\times -16 = 144 \qquad a=\frac{16\pm 12}{14} \longrightarrow a=\begin{cases}\cancel{2}\\ \frac{2}{7}\end{cases}$$

$$\frac{\frac{2}{7}+\frac{7}{7}}{\frac{2}{7}}=\frac{9}{2}=4.5$$

2

4ـ

$$\frac{\sqrt{x+1}}{\sqrt{x-1}+3}-\frac{\sqrt{x+1}}{3-\sqrt{x-1}}=\frac{x-1}{\sqrt{x-1}}$$

2

$$\frac{3\sqrt{x+1}-\sqrt{x^2-1}-\sqrt{x^2-1}-3\sqrt{x+1}}{9-(x-1)}=\frac{-2\sqrt{x^2-1}}{10-x}=\frac{x-1}{\sqrt{x-1}} \longrightarrow$$

$$-2\sqrt{x+1}\times(x-1)=(x-1)(10-x)$$

$$x-1>0 \longrightarrow x>1$$

$x \geq 10$

$-2\sqrt{x+1} = 10 - x \quad 4(x+1) = x^2 + 100 - 20x \longrightarrow x^2 - 24x + 96 = 0$

$\Delta = 8\sqrt{3}$

$$x = \frac{24 \pm 8\sqrt{3}}{2} = \begin{cases} \boxed{12 + 4\sqrt{3}} \checkmark \\ 12 - 4\sqrt{3} \; ✗ \end{cases}$$

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$$\frac{1}{\sqrt{2-x}+2} - \frac{1}{2-\sqrt{2-x}} = \frac{2-x}{5\sqrt{2-x}}$$

(۳)

$$\frac{1}{t+2} - \frac{1}{2-t} = \frac{2-2t-2}{2-t^2} = \frac{-2t}{4-t^2}$$

$$\frac{2-x}{5\sqrt{2-x}} = \frac{\cancel{t}^{2}}{5\cancel{t}} = \frac{t}{5} = \frac{-2t}{4-t^2} \longrightarrow -10 = t^2 - 4$$

$-10 = x + 2 \longrightarrow x = -12$ ریشه مثبت نداریم

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۶ ـ

$$\frac{1}{x^2} + \frac{1}{(1-x)^2} = \frac{160}{9} = \frac{(1-x)^2 + x^2}{x^2(1-x)^2} = \frac{16}{9}$$

(۳)

$$\frac{2x^2 - 2x + 1}{(x - x^2)^2} = \frac{160}{9} \longrightarrow \frac{2\underbrace{(x^2-x)}_{t}+1}{(x^2-x)^2} = \frac{160}{9}$$

$$\frac{2t+1}{t^2} = \frac{160}{9} = 160t^2 = 18t + 9 \longrightarrow 160t^2 - 18t - 9 = 0$$

$$t = \frac{18 \pm \sqrt{(-18)^2 - 4(160)(-9)}}{2 \times 160} = \frac{18 \pm 6\sqrt{9+160}}{2 \times 160 = 320}$$

$$t = \frac{18 \pm (6 \times 13)}{320} = t = \begin{cases} -\frac{3}{16} \longrightarrow x^2 - x = -\frac{3}{16} \longrightarrow x^2 - x + \frac{3}{16} = 0 \quad S_1 = 1 \\ \frac{3}{10} = x^2 - x = \frac{3}{10} \longrightarrow x^2 - x - \frac{3}{10} = 0 \quad S_2 = 1 \end{cases}$$

$x \neq 190 \qquad 1 + 1 = 2$

9-

$y = \sqrt{x^2 - 4x + 4} = |x-2|$

$y = \frac{1}{2}x + 2$

$x \geq 2 \qquad x - 2 = \frac{1}{2}x + 2 \longrightarrow 2x - 4 = x + 4 \longrightarrow x = 8$

$2 - x = \frac{1}{2}x + 2 \longrightarrow 4 - 2x = x + 4 \longrightarrow \begin{matrix} x = 2 \\ y = 2 \end{matrix}$

$$\begin{bmatrix} 2 & 5 & 1 \\ 0 & 3 & 1 \\ 8 & 6 & 1 \end{bmatrix} = \frac{1}{2} \times (16 + 12 + 0 - 4 - 0 - 0) = 12$$

10)

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{20} \Bigg\} \quad \frac{1}{x} + \frac{1}{x+9} = \frac{1}{20}$$

$x + 9 = y$

$\frac{x + x + 9}{x(x+9)} = \frac{1}{20} \to x^2 + 9x = 40x + 180$

$x^2 - 31x - 180 = 0 \qquad \Delta = 41$

$x = \frac{31 \pm 41}{2} = \frac{72}{2} = 36$

$-x^3+4x^2+25x-100 = -(x^3-4x^2-25x+100)$ (۷

$-(x-4)(x^2-25) = -(x-4)(x-5)(x+5) \geq 0$

$(x-4)(x-5)(x+5) \leq 0$ $\quad -5 \quad 4 \quad 5$ $\quad - + - +$ (۱) ۳

$x \in (-\infty, -5] \cup [4,5]$

$-x^2+6x-8 = -(x-2)(x-4) \geq 0 \longrightarrow (x-2)(x-4) \leq 0$ (۲)

$2 \quad 4$ $\quad - + -$ $\quad x \in [2,4]$

$(1),(2) \cap \longrightarrow \{4\}$

$\sqrt{4+\sqrt{0}} + \sqrt{16+\sqrt{0}} = 2+4 = 6$

$\boxed{x=4}$

$y = |x+2| + |x-1| \longrightarrow 2x+1 \quad x \geq 1$ (۸

$3 \quad -2 \leq x \leq 1$

$y = \dfrac{17-x}{3} \qquad -2x-1 \quad x \leq -2$ ۳

$x \leq -2 \longrightarrow -2x-1 = \dfrac{17-x}{3} \longrightarrow -6x-3 = 17-x \longrightarrow -5x = 20 \longrightarrow x=-4$

$y=7$

$A(-4,7)$

$2x+1 = \dfrac{17-x}{3} \longrightarrow 6x+3 = 17-x \longrightarrow 7x = 14 \quad x=2$

$y=5$

$B(2,5)$

$AB = \sqrt{(2+4)^2+(5-7)^2} = 2\sqrt{10}$

PILLNOTEBOOK