نام و نام خانوادگی: برنا کیانیان

تکلیف: ۱۸

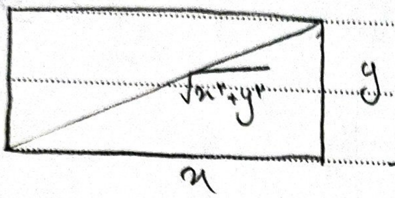
①

$\frac{L}{W} = \frac{5}{4} \longrightarrow L = 5x \ , \ W = 4x \to S = 20x^2$

$\frac{L'}{4x+W} = \frac{\sqrt5+1}{2} \Rightarrow L' = 2x(\sqrt5+1) \longrightarrow S' = 8x^2(\sqrt5+1) \longrightarrow$

$\frac{S'}{S} = \frac{8x^2(\sqrt5+1)}{20x^2} = \frac{2(\sqrt5+1)}{5}$

②



$\longrightarrow \frac{\sqrt{x^2+y^2}}{y} = \frac{\sqrt5+1}{2} \longrightarrow \frac{x^2+y^2}{y^2} = \frac{6+2\sqrt5}{4}$

$\longrightarrow \frac{x^2}{y^2}+1 = \frac{3+\sqrt5}{2} \longrightarrow \frac{y^2}{x^2} = \frac{2}{1+\sqrt5} \longrightarrow \frac{y^2}{x^2} = \frac{\sqrt5-1}{2}$

③

$\sqrt{2a^2+4a} = 2-3a \longrightarrow 2a^2+4a = 9a^2+4-12a \longrightarrow$

$7a^2-16a+4=0 \longrightarrow a^2-16a+28=0 \longrightarrow (a-14)(a-2)=0 \longrightarrow$

$a = 2$ غ ق ق ، $a = \frac{2}{7}$ ✓ $\longrightarrow \frac{a+1}{a} = \frac{\frac{9}{7}}{\frac{2}{7}} = \frac{9}{2}$

④

برای راحتی نوشتار $\sqrt[3]{x-1} = t \longrightarrow \frac{\sqrt{x+1}}{t+3} - \frac{\sqrt{x+1}}{3-t} = \frac{x-1}{t} \longrightarrow \frac{3\sqrt{x+1} - 2\sqrt{x^2-1} - 3\sqrt{x+1}}{10-x}$

$\longrightarrow \frac{-2\sqrt{x^2-1}}{10-x} = \frac{x-1}{t} \longrightarrow -2\sqrt{x+1}\,(t)^3 = t^3(10-x) \xrightarrow[x>1]{} $

$\longrightarrow -2\sqrt{x+1} = 10-x \xrightarrow[x\geq10]{\text{توان ۲}} 4x+4 = x^2+100-20x \longrightarrow$

$x^2-24x+96=0 \longrightarrow x = \frac{-b+\sqrt\Delta}{2a} \longrightarrow x = 12 \pm 4\sqrt3 \xrightarrow{x\geq10} 12+4\sqrt3$ ✓

⑤

$\sqrt{2-x} = t \longrightarrow \frac{1}{t+2} - \frac{1}{2-t} = \frac{t^2}{5t} \xrightarrow{x\neq2} \frac{-2t}{4-t^2} = \frac{t}{5} \longrightarrow$

$-10 = 4-t^2 \longrightarrow t^2-14=0 \longrightarrow x=-12 \longrightarrow$ ریشه ندارد

$x = 0.2t + t \rightarrow \frac{1}{(0.2t+t)^2} + \frac{1}{(0.2t-t)^2} = \frac{14}{9} \rightarrow$ (۹)

$\frac{0.25 + t^2 - t + 0.25 + t^2 + t}{(0.2t - t^2)^2} \rightarrow \frac{2(0.25 + t^2)}{(0.2t - t^2)^2} = \frac{140}{9} \rightarrow$

$324t^4 - 194t^2 + 11 = 0 \xrightarrow{t^2 = f} 324f^2 - 194f + 11 = 0 \rightarrow f = \frac{194 \pm 160}{648} \rightarrow$

$f = \frac{1}{18}, f = \frac{11}{20} \rightarrow t = \pm\frac{1}{4}, t = \pm\sqrt{\frac{11}{20}} \rightarrow x = 0.2t + t$

$x = \frac{1}{4}, x = \frac{3}{4}, x = \sqrt{\frac{11}{20}} + \frac{1}{2}, x = -\sqrt{\frac{11}{20}} + \frac{1}{2} \rightarrow$

$\frac{1}{4} + \frac{3}{4} + \sqrt{\frac{11}{20}} + \frac{1}{2} - \sqrt{\frac{11}{20}} + \frac{1}{2} = 2$

دامنه‌ها مشترک $\rightarrow$ ① $-x^3 + 4x^2 + 2x - 100 \geq 0 \rightarrow -(x-4)(x^2 - 25) \geq 0$ (۷)

می‌گیریم $\rightarrow$ $\begin{array}{ccccc} & -5 & & 4 & & 5 \\ + & \phi & - & \phi & + & \phi & - \end{array} \rightarrow x \in (-\infty, -5] \cup [4, 5]$

② $-x^2 + 6x - 8 \geq 0 \rightarrow -(x-2)(x-4) \geq 0 \rightarrow$ $\begin{array}{ccccc} & 2 & & 4 & \\ - & \phi & + & \phi & - \end{array}$ $x \in [2, 4]$

① و ② $\rightarrow x = 4 \rightarrow \sqrt{4+5} + \sqrt{16+5} = 2 + 4$ ✓ $\rightarrow x = 4 \rightarrow$ جواب

(۸)

$y = \frac{17 - x}{3} \rightarrow \frac{17 - x}{3} = |x+2| + |x-1| \xrightarrow{x \geq 1} 17 - x = 3(2x+1)$

$\rightarrow 7x = 14 \rightarrow x = 2 \rightarrow y = 5$ ، $x \leq -2 \rightarrow 3(-2x-1) = 17 - x$

$\rightarrow x = -4$ ، $y = 7$ $A(2,5)$ $B(-4,7)$ $\rightarrow AB = \sqrt{(2+4)^2 + (5-7)^2} = 2\sqrt{10}$

Arman

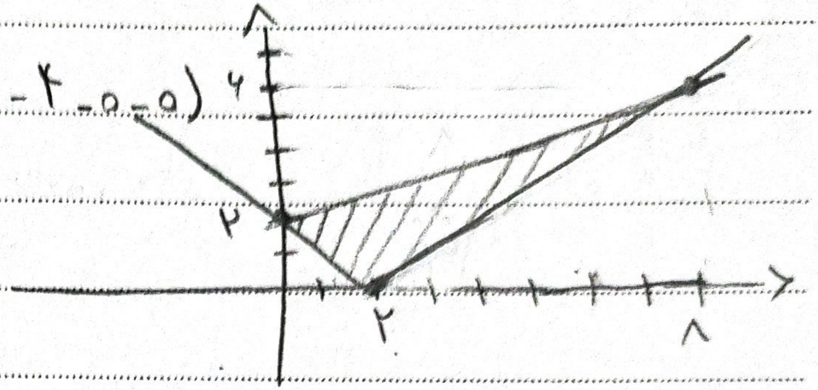
$\sqrt{x^۲-۴x+۴} = \sqrt{(x-۲)^۲} = |x-۲|$, $|x-۲| = \frac{۱}{۲}x+۲ \rightarrow$ (۹)

$\rightarrow x-۲ = \frac{۱}{۲}x+۲ \rightarrow \frac{۱}{۲}x = ۴ \rightarrow x=۸$, $y=۶$

$\rightarrow x-۲ = -\frac{۱}{۲}x-۲ \rightarrow \frac{۳}{۲}x = ۰ \rightarrow x=۰$, $y=۲$

$S_\triangle = \frac{۱}{۲} \times \begin{bmatrix} ۲ & ۰ & ۱ \\ ۰ & ۲ & ۱ \\ ۸ & ۶ & ۱ \end{bmatrix} = \frac{۱}{۲}(۱۴+۱۲+۰-۴-۰-۰)$

$S_\triangle = \frac{۱}{۲} \times ۲۴ = ۱۲$



عدد کوچکتر

$x = y-۹ \xrightarrow{\text{باهم}} \frac{۱}{x} + \frac{۱}{y} = \frac{۱}{۲۰} \rightarrow \frac{۱}{x} + \frac{۱}{x+۹} = \frac{۱}{۲۰}$ (۱۰)

$\frac{x+۹+x}{(x+۹)x} = \frac{۱}{۲۰} \rightarrow ۴۰x+۱۸۰ = x^۲+۹x \rightarrow x^۲-۳۱x-۱۸۰ \rightarrow$

$\rightarrow (x-۳۶)(x+۵) = ۰ \rightarrow x = ۳۶$