نام و نام خانوادگی: غزل عباس نژاد   کلاس: یازدهم گروه A   تکلیف: ۱۹

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$\alpha = 3\beta$

$S = \alpha+\beta = \frac{a}{3} \quad P = \alpha\cdot\beta = \frac{4}{3} \Rightarrow 3\beta\cdot\beta = 3\beta^2 = \frac{4}{3} \Rightarrow \beta^2 = \frac{4}{9}$

$\beta = \pm\frac{2}{3}$ ①

$\rightarrow 3\beta+\beta = \frac{a}{3} \Rightarrow 4\beta = \frac{a}{3} \quad 12\beta = a \xrightarrow{\text{جایگذاری ①}}$

$12\alpha\left(\mp\frac{2}{3}\right) = a \Rightarrow a = \pm 8 \qquad 8-(-8) = \boxed{16}$

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$\sqrt{\frac{1}{a}} + \sqrt{\frac{1}{\beta}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{\beta}} = \frac{\sqrt{\beta}+\sqrt{a}}{\sqrt{a\beta}} = 5 \Longrightarrow 5\times\frac{1}{6} = \sqrt{a}+\sqrt{\beta}$

$a\cdot\beta = \frac{1}{36}$

$a+\beta = \frac{m+14}{36} \xrightarrow{\text{به توان ۲}} \frac{25}{36} = a+\beta+2\sqrt{a\beta} \Rightarrow \frac{25}{36} = \frac{m+14}{36} + \frac{2}{6}$

$\xrightarrow{\times 36} 25 = m+14+12 \Rightarrow \boxed{m=-1} \qquad mx^2+3x+2 \Rightarrow P = \frac{2}{m} = \boxed{-2}$

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چند جمله‌ای مقابل بر $x^2-x-2=0$ بخش‌پذیر ـ خارج قسمت را $4x+m$ در نظر می‌گیریم

$(4x+m)(x^2-x-2) = 4x^3+kx^2-9x-2=0$

$4x^3+(m-4)x^2-(8+m)x-2m = 4x^3+kx^2-9x-2$

$m=1 \qquad m-4 = k \xrightarrow{m=1} \boxed{k=-3}$

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$a+\beta = -6 \qquad 2(a^2+\beta^2)+a^2 = 12\sqrt{2}+85 \Rightarrow 2\left((\cancel{a+\beta})^2 - 2a\beta\right) =$

$12\sqrt{2}+85 \Rightarrow 72 - 4a\beta = 12\sqrt{2}+85 \Rightarrow 72-85-12\sqrt{2} = 4a\beta$

$-12\sqrt{2}-13 = 4a\beta \Rightarrow \cancel{a\beta} = -3\sqrt{2} - \frac{13}{4}$

$\alpha^2+6\alpha-3\sqrt{2}-\frac{13}{4} = 0 \Rightarrow \alpha = \frac{-6\pm\sqrt{36+12\sqrt{2}+13}}{2}$

$\frac{-6\pm\sqrt{49+12\sqrt{2}}}{2} \Rightarrow a<0 \quad a = \frac{-6+\sqrt{49+12\sqrt{2}}}{2}$

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$x^4-7x^2-5x=0 \xrightarrow{\text{ریشه}} x_1^2 = \frac{+7\pm\sqrt{\cancel{49+20}}}{2} \Rightarrow x_1 = \sqrt{\frac{7+\sqrt{69}}{2}}$

$x_2 = -\sqrt{\frac{+7+\sqrt{69}}{2}} \qquad S=0 \quad P = -\frac{\sqrt{7+\sqrt{69}}}{\sqrt{2}} \times \sqrt{\frac{7+\sqrt{69}}{2}} =$

$-\frac{49+69+14\sqrt{69}}{4} = P$

$2P^2-3SP+2S \xrightarrow{S=0} \cancel{2}P^2 = \left(\frac{-49-69-14\sqrt{69}}{2}\right)^2 = 59+7\sqrt{69}$

$2x^2 - ax + b = 0 \Longrightarrow x_1 + 0/5 \,/\, x_2 + 0/5 \Longrightarrow S = x_1 + x_2 + 1 = \frac{a}{2}$ ① 6

$2ax^2 + ax - 6 = 0 \Longrightarrow x_1 \,/\, x_2 \Longrightarrow x_1 + x_2 = S = \frac{-1}{2}$

$\xrightarrow{①} \frac{-1}{2} + 1 = \frac{a}{2} \to a = 1 \qquad x_1 \cdot x_2 = \frac{-3}{a} \;, = -3$

$(x_1 + \frac{1}{2})(x_2 + \frac{1}{2}) = \frac{b}{2} \to \overset{-3}{\cancel{x_1 x_2}} + \frac{1}{2}x_1 + \frac{1}{2}x_2 + \frac{1}{4} = \frac{b}{2}$

$-3 + \frac{1}{2}(\underbrace{\cancel{x_1 + x_2}}_{\frac{-1}{2}}) + \frac{1}{4} = \frac{b}{2} \Longrightarrow -3 - \frac{1}{4} + \frac{1}{4} = \frac{b}{2}$

$b = -6 \qquad \left[\frac{ab}{4}\right] = \left[\frac{-6 \times 1}{4}\right] = \left[-\frac{3}{2}\right] = -2$

$ax^2 - ax - b = 0 \qquad 40\beta^2 + 20\alpha^2 - 20\beta = 17$ 7

$\hookrightarrow \alpha + \beta = 1 \qquad \hookrightarrow 20\beta^2 + 20\alpha^2 + 20\beta^2 - 20\beta = 17$

$\alpha \cdot \beta = \frac{-b}{a} \qquad 20\left[(\beta^2 + \alpha^2) + \beta^2 - \beta\right] = 17$

$20\left[(\overset{1}{\cancel{\alpha + \beta}})^2 - 2\alpha\beta\right] + 20(\beta^2 - \beta) = 17 \Longrightarrow 20\left(+1 + \frac{2b}{a}\right) +$

$20\beta(\beta - 1) = 17 \Longrightarrow 20 + \frac{40b}{a} + 20\beta^2 - 20\beta = 17$

$20\beta^2 - 20\beta = \overset{-3}{\cancel{17 - 20}} - \frac{40b}{a} = -\left(3 + \frac{40b}{a}\right) \xrightarrow{\text{جایگذاری}} 20 + \frac{40b}{a} - 3 - \frac{40b}{a}$

$= 17$ ✓

راه سریع‌تر:

$a - 1 = \beta \xrightarrow{\text{جایگذاری در عبارت}} 40(a-1)^2 + 20a^2 - 20(a-1) = 17$

$\Longrightarrow 60a^2 - 60a + 3 = 0 \Longrightarrow 20a^2 - 20a + 1 = 0$

$\frac{\sqrt{\Delta}}{a} = \frac{\sqrt{400 - 80}}{20} = \frac{\sqrt{52}}{5}$

$x^2 - (a+1)x + a = 0 \Longrightarrow x_1 + x_1 + 2 = a + 1 \Longrightarrow 2x_1 = a - 1$ 8

$\Longrightarrow P_1 = x_1^2 + 2x_1 = a \qquad \frac{a}{2} \in Z \qquad ① \; x_1 = \frac{a-1}{2}$

$x^2 - (3a+1)x + b = 0 \qquad x_2 + x_2 + 2 = 3a + 1 \Longrightarrow x_2 = \frac{3a-1}{2}$ ②

$\Longrightarrow P_2 = x_2^2 + 2x_2 = b$

① $2x_1 + 1 = a \;,\; x_1^2 + 2x_1 = a \Longrightarrow \cancel{2x_1} + x_1^2 = \cancel{2x_1} + 1 \Longrightarrow x_1 = \pm 1$

$\Longrightarrow +1$ ✓

$x_1 = 1 \;,\; x_1 + 2 = 3 \;,\; a = 1 + 2 = 3$

$x_2 = \frac{3a-1}{2} = \frac{3 \times 3 - 1}{2} = 4 \qquad x_2 + 2 = 6$

$|P_1 - P_2| = |3 - 24| = \boxed{21}$

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$\alpha + \beta = -۱ \qquad \alpha \cdot \beta = -(m^{۲}+۱) \qquad \alpha^{۲}+\beta^{۲} = (\alpha+\beta)^{۲} - ۲\alpha\beta$

$= ۱ + ۲m^{۲} + ۲ = ۲m^{۲} + ۳ \longrightarrow \min \quad y_s = \frac{-\Delta}{۴a} = \frac{-(-۲۴)}{۸} = \boxed{۳}$

۱۰

$\alpha^{۲}+\beta^{۲} = ۵ \Longrightarrow (\alpha+\beta)^{۲} - ۲\alpha\beta = ۵ \quad ①$

$y = ax^{۲} + bx + c \Longrightarrow \underset{\text{دو نقطه برابر}}{y} \Longrightarrow ۲۵a - ۵b + c = ۹a + ۳b + c$

$\Longrightarrow ۱۶a = ۸b \qquad ۲a = b \qquad y_s = ۱ = \frac{-\Delta}{۴a} = \frac{-b^{۲}+۴ac}{۴a} = ۱$

$\xrightarrow{b=۲a} \frac{-۴a^{۲}+۴ac}{۴a} = ۱ \Longrightarrow ۴a = -۴a^{۲} + ۴ac \Longrightarrow$

$\cancel{۴a} = \cancel{۴a}(c-a) \Longrightarrow c - a = ۱ \qquad \boxed{c = a+۱} \qquad c = \frac{b}{۲} + ۱$

$\alpha + \beta = \frac{-b}{a} \qquad \alpha\beta = \frac{c}{a} \xrightarrow{①} \left(\frac{-b}{a}\right)^{۲} - ۲\left(\frac{c}{a}\right) = ۵$

$\xrightarrow[۲a = b]{c = a+۱ = \frac{b}{۲}+۱} \left(\frac{-۲\cancel{a}}{\cancel{a}}\right)^{۲} - ۲\left(\frac{a+۱}{a}\right) = ۵ \Longrightarrow ۴ - \left(\frac{۲a+۲}{a}\right) = ۵$

$\frac{۴a - ۲a + ۲}{a} = ۵ \Longrightarrow ۵a = \cancel{۴a} - ۲a + ۲ \qquad ۳a = ۲$

$a = \frac{۲}{۳}$

$c = \frac{۲}{۳} + ۱ = \frac{۵}{۳}$