①

$\alpha = 2\beta$ $\quad 2x^2 - ax + 6 = 0$ $\quad \alpha\beta = \frac{c}{a} \Rightarrow 2\beta\beta = \frac{6}{2}$

$2\beta^2 = 3 \Rightarrow \beta^2 = \frac{3}{2} \Rightarrow \beta = \pm\frac{\sqrt{3}}{2}$ ، $\alpha = \pm\sqrt{2}$

$\alpha+\beta = -\frac{b}{a} \Rightarrow 3\beta = \frac{a}{2} \Rightarrow 12\beta = a \Rightarrow \sqrt{2}(\frac{\sqrt{3}}{2}) = a$

$a_1 = 8$

$12(-\frac{1}{2}) = a \Rightarrow a_2 = -8$

$\Rightarrow a_1 - a_2 = 8 - (-8) = 16$

$a_2 - a_1 = -8 - 8 = -16$

②

$24x^2 - (m+14)x + 1 = 0$ $\quad \frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 0 \rightarrow \frac{\sqrt{\beta}+\sqrt{\alpha}}{\sqrt{\alpha\beta}} = 0 \Rightarrow \frac{\sqrt{\beta}+\sqrt{\alpha}}{\frac{1}{4}} = 0 \Rightarrow \sqrt{\beta}+\sqrt{\alpha} = \frac{0}{4}$

$\alpha\beta = \frac{c}{a} = \frac{1}{24}$

$\sqrt{\alpha\beta} \rightarrow \sqrt{\frac{1}{24}} = \pm\frac{1}{4}$ (غ ق ق)

$\alpha+\beta = -\frac{b}{a} = \frac{m+14}{24}$ $\quad (\frac{a}{2})^2 - 2(\frac{1}{2}) = \frac{12}{24} - \frac{2}{4} = \frac{25}{24} - \frac{12}{24} = \frac{13}{24} \Rightarrow \frac{m+14}{24} = \frac{13}{24} \Rightarrow m = -1$

$m=-1 \rightarrow -x^2 + 4x + 2 = 0 \rightarrow P = \frac{c}{a} = \frac{2}{-1} = -2$

④

$\alpha\beta = 2$
$\alpha+\beta = 1$

$2x^3 + kx^2 - 9x - 2 = 0$

$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta \Rightarrow 1 = \alpha^2 + \beta^2 - 4 \Rightarrow \alpha^2 + \beta^2 = 5$

$(\alpha-\beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta = 5 + 4 = 9 \Rightarrow \alpha - \beta = 3$ ، $\alpha + \beta = 1$ $\Rightarrow \alpha = 2$ ، $\beta = -1$

$-4 + k + 9 - 2 = 0 \Rightarrow k = -3$

⑤

$a^2 + 2\sqrt{2}a + a = 0$

$\alpha < \beta < 0$ ، $4\alpha^2 + 2\beta^2 = 12\sqrt{2} + 18$

$2(\alpha^2+\beta^2) + \alpha^2 = 12\sqrt{2} + 18 \Rightarrow 2(18 - a - 4\sqrt{9-a}) + 18 - a + 4\sqrt{9-a}$

$\Delta = 32 - 4a \rightarrow \alpha, \beta = \frac{-4 \pm 4\sqrt{9-a}}{2}$ $\overset{\beta > \alpha}{\Rightarrow}$ $\beta = -2 + 2\sqrt{9-a}$

$a = -2 - \sqrt{9-a}$

$\beta^2 = 9 + 9 - a - 4\sqrt{9-a}$ ، $\alpha^2 = 9 + 9 - a + 4\sqrt{9-a}$

$12\sqrt{2} - 5 = -5a + 4\sqrt{9-a} \Rightarrow -5a = -5$

$a = 1$

⑥

$x^4 - 7x^2 - 5 = 0 \rightarrow t^2 - 7t - 5 = 0$ $\quad 49 - 4(1)(-5) = 69 \Rightarrow t = \frac{7 \pm \sqrt{69}}{2}$

$t = x^2 \rightarrow x^2 \neq \frac{7-\sqrt{69}}{2}$ منفی $\Rightarrow x^2 = \frac{7+\sqrt{69}}{2} \Rightarrow x = \pm\sqrt{\frac{7+\sqrt{69}}{2}}$

$x_1 + x_2 = 0$

$x_1 \times x_2 = +\sqrt{\frac{7+\sqrt{69}}{2}} \times -\sqrt{\frac{7+\sqrt{69}}{2}} \rightarrow P^2 = 2\left(\frac{7+\sqrt{69}}{2}\right)^2 = 2\left(\frac{49+69+14\sqrt{69}}{4}\right) = 59 + 7\sqrt{69}$

⑦

$2x^2 - ax + b = 0$ $\quad \alpha + \beta = \frac{a}{2}$ $\quad \alpha\beta = \frac{b}{2}$

$4ax^2 + ax - 6 = 0$ $\quad \alpha' + \beta' = -\frac{1}{4}$ $\quad \alpha'\beta' = -\frac{3}{a}$

$(\alpha - 0.5) + (\beta - 0.5) = \alpha + \beta - 1 = -\frac{1}{2} \Rightarrow \frac{a}{2} = \frac{1}{2} \Rightarrow a = 1$

$(\alpha - 0.5)(\beta - 0.5) = \alpha\beta - 0.5\alpha - 0.5\beta + 0.25 = -4$

$\frac{b}{2} - 0.5(\alpha+\beta) = \frac{b}{2} - \frac{a}{4}$

$\frac{b}{2} = -3 \Rightarrow b = -6$

$\left[\frac{a \times b}{6}\right] = \left[\frac{1 \times (-6)}{6}\right] = -2$

اگر برابر صفر باشد

$ax^2 - ax - b = 0 \qquad \alpha + \beta = 1 \rightarrow \alpha = 1 - \beta$ ①

$4\beta^2 + 20(1-\beta)^2 - 20\beta = 17 \rightarrow 40\beta^2 - 40\beta + 3 = 0$

$20\beta^2 - 20\beta + 1 = 0 \rightarrow \beta = \frac{20 \pm \sqrt{240}}{40} = \frac{5 \pm 2\sqrt{5}}{10} \Rightarrow \alpha = 1 - \beta \Rightarrow \alpha = 1 - \frac{5 \pm 2\sqrt{5}}{10} \Rightarrow$

$\alpha = \frac{5 \pm 2\sqrt{5}}{10}$

$\alpha - \beta = \frac{4\sqrt{5}}{10} = \frac{2\sqrt{5}}{5}$

$x^2 - (a+1)x + a = 0$ معادله دوم $\rightarrow a + b + c = 0 \Rightarrow x = 1, a = 1, 2 \Rightarrow a = 2$

$x^2 - (2a+1)x + b = 0$ معادله سوم $\rightarrow a = 2 \quad 2x^2 - 10x + b = 0 \rightarrow S = 10, P = b, \alpha = \beta + 2$ ④

$\alpha + \beta = 10 \rightarrow \beta = 4, \alpha = 6$

$P_2 - P_1 = 24 - 3 = 21 \Leftarrow P_2 = \alpha\beta = 6 \times 4 = 24$

$x^2 + x + (-m^2 - 1) = 0 \qquad \alpha\beta = -m^2 - 1, \ \alpha + \beta = -1 \Rightarrow \min = S^2 - 2P = 1 - 2P$ ⑦

$\left.\begin{matrix} 1 + 2m^2 + 2 = 2m^2 + 3 \\ m = 0 \end{matrix}\right\} \Rightarrow 3 =$ کمترین مقدار

$x_S = \frac{-a}{2} = -1, \ y_S = 1 \Rightarrow y = a(x+1)^2 + 1 = ax^2 + 2ax + a + 1$ ⑩

$A\binom{x}{y} \quad B\binom{-x}{y}$

$\alpha^2 + \beta^2 = 5 \Rightarrow \left(\frac{-2a}{a}\right)^2 - 2\left(\frac{a+1}{a}\right) = 5 \Rightarrow \frac{-2a-2}{a} = 1 \Rightarrow -2a - 2 = a$

$a = -\frac{2}{3}$

$y = -\frac{2}{3}x^2 - \frac{4}{3}x + \left(\frac{-2}{3} + 1\right) \xrightarrow{x=0} y = \frac{1}{3}$