

$$\begin{cases} \alpha = 3\beta \\ \alpha\beta = \frac{5}{4} \end{cases} \Rightarrow 3\beta \times \beta = \frac{5}{4} \Rightarrow \beta^2 = \frac{5}{4} \Rightarrow \beta = \pm \frac{\sqrt{5}}{2} \Rightarrow \alpha = \pm \sqrt{5}$$

(1)

$$\alpha + \beta = \frac{a}{4} \Rightarrow \begin{cases} 3 + \frac{\sqrt{5}}{2} = \frac{a}{4} \Rightarrow \frac{1}{4} = \frac{a}{4} \Rightarrow a = 1 \\ (-1) + (-\frac{\sqrt{5}}{2}) = \frac{a}{4} \Rightarrow a = -1 \end{cases} \Rightarrow 1 - (-1) = 14$$

$$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = a \Rightarrow \frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = a \Rightarrow \frac{1}{\alpha} + \frac{1}{\beta} + \frac{2}{\sqrt{\alpha\beta}} = 2a \Rightarrow \frac{\alpha + \beta}{\alpha\beta} + \frac{2}{\sqrt{\alpha\beta}} = 2a$$

(2)

$$\Rightarrow \frac{m+1}{\frac{1}{\sqrt{5}}} + \frac{2}{\sqrt{\frac{5}{4}}} = 2a \Rightarrow m = -1 \quad m\alpha^2 + 3\alpha + 1 = 0 \Rightarrow -\alpha^2 + 3\alpha + 1 = 0 \Rightarrow \frac{c}{a} = \frac{1}{-1} = -1$$

$$S = \alpha + \beta, P = \alpha\beta \Rightarrow S = 1, P = -2$$

(3)

$$x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0 \Rightarrow x = -1, 2$$

$$x = -1 \Rightarrow -x + k + 9 - 2 = 0 \Rightarrow k = -3$$

$$3\alpha^2 + 2\beta^2 \Rightarrow \alpha^2 + 2\beta^2 + 2\alpha^2 \rightarrow \alpha^2 + \beta^2 = 5^2 - 2P \Rightarrow \alpha^2 + \beta^2 = 25 - 2(-2) = 29$$

$$\Delta = 29 - 4a \quad \alpha = \frac{-5 \pm \sqrt{29 - 4a}}{2} \Rightarrow \alpha^2 = \frac{25 \pm 5\sqrt{29 - 4a} + 29 - 4a}{4} \Rightarrow \alpha^2 = \frac{54 \pm 5\sqrt{29 - 4a} - 4a}{4}$$

$$\frac{12\sqrt{29 - 4a}}{4} = 12\sqrt{2} \Rightarrow a = 1$$

$$x^2 = t \quad x^2 - \sqrt{2}x - a = 0 \Rightarrow \text{جميع الجذور حقيقية}$$

$$t^2 - \sqrt{2}t - a = 0 \Rightarrow t = \frac{\sqrt{2} \pm \sqrt{2 + 4a}}{2}, t = \frac{\sqrt{2} - \sqrt{2 + 4a}}{2} \rightarrow \text{ق ق } t = x^2$$

(4)

$$\Rightarrow x^2 = \frac{\sqrt{2} \pm \sqrt{2 + 4a}}{2} \Rightarrow \begin{cases} x_1 = \sqrt{\frac{\sqrt{2} + \sqrt{2 + 4a}}{2}} \\ x_2 = \sqrt{\frac{\sqrt{2} - \sqrt{2 + 4a}}{2}} \end{cases} \Rightarrow P = x_1 x_2 = -\sqrt{\frac{(2 + 4a)}{4}}$$

$$\Rightarrow 2P^2 = 2 \left(\frac{(2 + 4a)}{4} \right) = \frac{(2 + 4a)^2}{4} = \frac{4 + 16a + 16a^2}{4} = 1 + 4a + 4a^2 = 5 + 4\sqrt{29}$$

$$\alpha + \beta = \alpha' + \beta' \Rightarrow \frac{\alpha}{4} = \frac{-\alpha}{4} + 1 = \frac{1}{4} \Rightarrow \alpha = 1$$

(5)

$$\Rightarrow 2x^2 + x - 6 = 0 \Rightarrow \alpha', \beta' = -2, \frac{3}{2} \Rightarrow \alpha, \beta = \frac{-x}{4}, 2$$

$$\Rightarrow \frac{b}{4} = \alpha\beta = -2 \Rightarrow b = -8$$

$$\left[\frac{ab}{4} \right] = \left[\frac{-8}{4} \right] = -2$$

$$2\alpha\beta^2 + 2\alpha^2\beta + 2\alpha\beta^2 - 2\alpha\beta = 14 \Rightarrow 2\alpha(\alpha^2 + \beta^2) + 2\alpha(\beta^2 - \beta) = 14 \Rightarrow 2\alpha(5^2 - 2P) + 2\alpha(\beta^2 - \beta) = 14$$

(6)

$$S = \frac{-(-a)}{a} = 1, P = \frac{-b}{a} \quad \alpha\beta^2 - \alpha\beta - b = 0 \Rightarrow \alpha(\beta^2 - \beta) = b \Rightarrow \beta^2 - \beta = \frac{b}{\alpha}$$

$$2\alpha(5^2 - 2P) + 2\alpha(\beta^2 - \beta) = 14 \Rightarrow 2\alpha(1 + \frac{b}{\alpha}) = 14 \Rightarrow 2\alpha + \frac{2b}{\alpha} = 14 \Rightarrow \frac{2b}{\alpha} = 12 \Rightarrow \frac{b}{\alpha} = \frac{6}{1}$$

$$\frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{(-a)^2 - 4a(-b)}}{|a|} = \frac{\sqrt{a^2 + 4ab}}{|a|} = \frac{|a|\sqrt{1 + \frac{4b}{a}}}{|a|} = \sqrt{1 + 4\left(\frac{1}{1}\right)} = \frac{2}{\sqrt{a}}$$

$$x^2 - (a+1)x + a = 0 \Rightarrow \text{مجموع جذور = صفر اولی} \Rightarrow \begin{cases} x_1 = 1 & \text{رابطه ها در اولی} \\ x_2 = a & \text{فرض دومین اند} \end{cases} a = 3$$

⑧

$$x^2 - (3a+1)x + b = 0 \xrightarrow{a=3} x^2 - 10x + b = 0 \Rightarrow x_1 + x_2 = 10 \xrightarrow{\text{زوج } x_2 \text{ و } x_1 \text{ زوج}} x_1 = 2, x_2 = 8$$

$$\text{مجموع حاصل ضرب در اولی} = 14 - 3 = 11$$

$$\alpha^2 + \beta^2 = s^2 - 2p \quad s = \frac{-1}{1} = -1 \quad p = \frac{-1-m^2}{1} = -1-m^2$$

$$\alpha^2 + \beta^2 = (-1)^2 - 2(-1-m^2) = 1 + 2 + 2m^2 = 2m^2 + 3 \Rightarrow \text{مجموع مربعات = 3} \rightarrow \text{مجموع مربعات = 3}$$

⑨

$$x_5 = \frac{-a+3}{2} = \frac{-2}{2} = -1 \quad s(-1, 1) \Rightarrow y = a(x+1)^2 + 1 \Rightarrow y = ax^2 + 2ax + a + 1$$

$$\alpha^2 + \beta^2 = a \Rightarrow s^2 - 2p = a \Rightarrow \left(-\frac{2a}{a}\right)^2 - 2\left(\frac{a+1}{a}\right) = a \Rightarrow 4 - 2\left(\frac{a+1}{a}\right) = a \Rightarrow \frac{a+1}{a} = \frac{-1}{2}$$

⑩

$$\Rightarrow -a = 2a + 2 \Rightarrow a = -\frac{2}{3} \quad \alpha + \beta \xrightarrow{\text{مجموع جذور = صفر اولی}} \omega y \Rightarrow \frac{-2}{3} + \frac{2}{3} = \frac{0}{3}$$