۱-

$3x,x \Rightarrow \quad 3x^2=\frac{1}{3} \Rightarrow 9x^2=1 \Rightarrow x^2=\frac{1}{9} \Rightarrow x=\pm\frac{1}{3}$

$4x+\frac{a}{3}=12x+a \begin{cases} 12\times -\frac{2}{3}=-8 \\ 12\times \frac{2}{3}=8 \end{cases} \quad 8-(-8)=\boxed{16}$

۲-

$\sqrt{\frac{1}{x}}+\sqrt{\frac{1}{y}}=5 \Rightarrow \frac{1}{x}+\frac{1}{y}+2\sqrt{\frac{1}{xy}}=25 \Rightarrow \frac{x+y}{xy}+2\sqrt{\frac{1}{xy}}=25$

$x+y=\frac{m+14}{36}, \quad xy=\frac{1}{36}$

$\frac{\frac{m+14}{36}}{\frac{1}{36}}+2\sqrt{36}=25$

$mx^2+3x+2=0 \xrightarrow{m=-1} -x^2+3x+2=0 \Leftarrow m+14+12=25 \Rightarrow m=-1$

ضرب $=\frac{c}{a}=\frac{2}{-1}=\boxed{-2}$

۳-

$(a+b)^2=a^2+b^2+2ab$

$1=a^2+b^2-4 \Rightarrow a^2+b^2=5$ $\quad \begin{cases} a+b=1 \\ a-b=-3 \end{cases} \Rightarrow b=2, \ a=-1$

$(a-b)^2=a^2+b^2-2ab$

$(a-b)^2=5+4 \Rightarrow a-b=\pm 3$ $\quad \begin{cases} a+b=1 \\ a-b=+3 \end{cases} \Rightarrow a=2, \ b=-1$

جایگذاری یکی از ریشه‌ها $\Rightarrow 4(2)^3+k(2)^2-9(2)-2=0 \Rightarrow \boxed{k=-3}$

۴-

$a+b=-6 \quad aB=c \quad a<B<0 \quad 3\alpha^2+2\beta^2=12\sqrt{2}+85 \Rightarrow \frac{5}{2}(\alpha^2+\beta^2)+\frac{1}{2}(\alpha^2-\beta^2) \Rightarrow \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\Rightarrow 36-2\alpha\beta$

$\alpha^2-\beta^2=(\alpha+\beta)(\alpha-\beta) \Rightarrow -6(\sqrt{\Delta})$

$12\sqrt{2}+85=\frac{5}{2}(36-2\alpha\beta)+\frac{1}{2}(-6\sqrt{\Delta}) \Rightarrow 24\sqrt{2}+170=5(36-2\alpha\beta)-6\sqrt{\Delta}$

$\Rightarrow 24\sqrt{2}=-6\sqrt{\Delta} \to \sqrt{\Delta}=-4\sqrt{2}$

$\frac{170}{5}=36-2\alpha\beta \to 34-36=-2\alpha\beta \Rightarrow 2\alpha\beta=2 \Rightarrow \alpha\beta=1 \Rightarrow \frac{c}{a}=\frac{c}{1} \Rightarrow c=1$

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$\boxed{a=1}$

۵-

$x^4-7x^2-5=0 \xrightarrow{x^2=t} t^2-7t+5=0$

$t=\frac{7\pm\sqrt{49+20}}{2} \quad t>0 \Rightarrow t=\frac{7+\sqrt{69}}{2} \Rightarrow x=\pm\sqrt{\frac{7+\sqrt{69}}{2}}$

$\Rightarrow S=0$

$P=\frac{-7-\sqrt{69}}{2}$

$2P^2-\cancel{3SP}+\cancel{2S}=2P=2\times\left(\frac{-7-\sqrt{69}}{2}\right)^2=59+7\sqrt{69}$

$2ax^2 + ax - 6 = 0 \longrightarrow \begin{cases} T_1 + 0/5 = S_1 \\ T_2 + 0/5 = S_2 \end{cases}$; $S_1 + S_2 = 1 + T_1 + T_2$ ۶-

$\frac{-a}{2a} = -\frac{1}{2} = T_1 + T_2$; $1 - \frac{1}{2} = S_1 + S_2 \Rightarrow \frac{1}{2} = S_1 + S_2 \Rightarrow \frac{a}{2} = \frac{1}{2} \Rightarrow a = 1$

$-\frac{b}{2} = (T_1 + 0/5)(T_2 + 0/5) = T_1T_2 + \frac{T_1 + T_2}{2} + \frac{1}{4} = -3 \Rightarrow T_1T_2 - \frac{1}{4} + \frac{1}{4} = -3 \Rightarrow T_1T_2 = -3$

$\Rightarrow \frac{b}{2} = -3 \Rightarrow b = -6$

$\left[\frac{-6 \times 1}{4}\right] = \left[\frac{-6}{4}\right] = \left[\frac{-3}{2}\right] = \boxed{-2}$

$ax^2 - ax - b = 0$; $40\beta^2 + 20\alpha^2 - 20\alpha = 17$ ۷.

$\hookrightarrow \alpha^2 = \frac{a\alpha + b}{a} \rightarrow \beta^2 = \frac{a\beta + b}{a}$

$40 \times \frac{a\beta + b}{a} + 20 \times \frac{a\alpha + b}{a} - 20\beta = 17 \Rightarrow 40a\beta + 40b + 20a\alpha + 20b - 20\beta a = 17a$

$\Rightarrow 20a + 40b = 17a \Rightarrow 40b = -3a \Rightarrow b = -\frac{3a}{40}$

$\frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{a^2 + 4a \times \frac{-3}{40}a}}{|a|} = \frac{\sqrt{\frac{7}{10}a^2}}{|a|} = \sqrt{\frac{7}{10}}$

$\begin{cases} S = a + 1 = 2\alpha + 2 \\ P = a = \alpha^2 + 2\alpha \end{cases}$; $\alpha^2 + 2\alpha + 1 = 2\alpha + 2 \Rightarrow \alpha^2 = 1 \rightarrow \begin{cases} \alpha = \pm 1 \\ \alpha \in N \end{cases} \Rightarrow a = 1 + 2 = 3$ ۸-

$x^2 - (\underbrace{3a+1}_{10})x + b = 0 \longrightarrow \beta, \beta + 2$

$S = 10 = 2\beta + 2 \longrightarrow \beta = 4$

$b = 4 \times 6 = 24$

$b - a = 24 - 3 = \boxed{21}$

$\begin{cases} \alpha + \beta = \frac{-b}{a} = -1 \\ ab = -1 - m^2 \end{cases} \rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 - 2(-1 - m^2) = 2m^2 + 3 \xrightarrow{m^2 \geq 0} \boxed{\min = 3}$ ۹-

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$A(3, y)$ $B(-5, y)$ $x_s = \frac{-5 + 3}{2} = -1$, $y = 1$ $r = \sqrt{-\frac{1}{a}}$ $x = -1 \pm r$ ۱۰

$y - 1 = a(x + 1)^2 \Rightarrow 0 - 1 = a(x + 1)^2 \Rightarrow (x + 1)^2 = -\frac{1}{a}$ $\alpha = -1 + r$ $\beta = -1 - r$

$(\alpha^2 + \beta^2) = 5 \longrightarrow (-1 + r)^2 + (-1 - r)^2 \Rightarrow (1 - 2r + r^2) + (1 + 2r + r^2) = 2 + 2r^2$

$\Rightarrow 2r^2 + 2 = 5 \rightarrow r^2 = \frac{3}{2}$ $-\frac{1}{a} = \frac{3}{2} \rightarrow a = -\frac{2}{3}$ $x = 0 \leftarrow$ عرض از مبدا

$y - 1 = a(0 + 1)^2 = a \Rightarrow$

$y = 1 + a = 1 - \frac{2}{3} = \boxed{\frac{1}{3}} \longrightarrow$ عرض از مبدا