نام: تکلیف شماره ۱۹ پایه: دهم A

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۱)

$3x^2 - ax + 4 = 0$ $x_1 = 3x_2 \Rightarrow x_1 \cdot x_2 = \frac{c}{a} \Rightarrow$

$3x_2 \cdot x_2 = \frac{4}{3} \to x_2^2 = \frac{4}{9} \to x_2 = \pm\frac{2}{3}$

$x_2 = \frac{2}{3} \to x_1 = 3\times\frac{2}{3} = 2 \to x_1+x_2 = \frac{8}{3} \to \frac{8}{3} = \frac{a}{3} \to a = 8$

$x_2 = \frac{-2}{3} \to x_1 = 3\times\frac{-2}{3} = -2 \to x_1+x_2 = -\frac{8}{3} \to -\frac{8}{3} = \frac{a}{3} \to a = -8$

$\to 8-(-8) = 16$

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۲)

$36x^2 - (m+14)x + 1 = 0$ $\left(\sqrt{\frac{1}{x_1}} + \sqrt{\frac{1}{x_2}}\right)^2 = (5)^2$ توان ۲

$\frac{1}{x_1} + \frac{1}{x_2} + 2\sqrt{\frac{1}{x_1x_2}} = 25 \to \frac{x_2+x_1}{x_1x_2} + 2\sqrt{\frac{1}{x_1x_2}} = 25 \to$

$x_1 \cdot x_2 = \frac{c}{a} = \frac{1}{36} \Rightarrow 36(x_1+x_2) + 2\sqrt{36} = 25 \to x_1+x_2 = \frac{13}{36}$

$x_1+x_2 = \frac{m+14}{36} = \frac{13}{36} \to m+14 = 13 \to m = -1$

$mx^2 + 3x + 2 = 0 \xrightarrow{m=-1} -x^2 + 3x + 2 = 0 \to \alpha\cdot\beta = -2$

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۳)

$4x^3 + Kx^2 - 9x - 2 = 0$, $\alpha+\beta = 1$, $\alpha\cdot\beta = -2$ $K = ?$

$\beta = 1-\alpha$

$\alpha(1-\alpha) = -2 \to \alpha = -1, 2$

$\alpha = 2 \to K = -3$

$\alpha = -1 \to K = -3$

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۴)

$x^2 + 6x + a = 0$ $\alpha < \beta < 0$

$3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85$ $a = ?$

$\alpha+\beta = -6$, $\alpha\cdot\beta = a$ $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$2\alpha^2 + 2\beta^2 + \alpha^2 = 12\sqrt{2} + 85 \to 2(\alpha^2+\beta^2) + \alpha^2 = 2(36-2a) + \alpha^2 = 72 - 4a + \alpha^2$

$\to \alpha^2 - 4a + 72 = 12\sqrt{2} + 85$

$S = -6$ $\alpha\beta = a$ $\alpha,\beta = \frac{-6\pm\sqrt{36-4a}}{2} \to \alpha < \beta \to \alpha = -3-\sqrt{9-a}$ (۴

$3\alpha^2 + 2\beta^2 = \alpha^2 + 2(S^2 - 2P) = 12\sqrt{2} + 85 \to$

$90 - 5a = 85 \to a = 1$

$6\sqrt{9-a} = 12\sqrt{2} \xrightarrow{a=1} 6\sqrt{9-a} = 6\sqrt{8} = 12\sqrt{2} = 1$

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$x^4 - 7x^2 - 8 = 0 \qquad x^2 = t$

$t^2 - 7t - 8 = 0 \rightarrow S = 7 \;,\; P = -8$

$t^2 - 7t - 8 = 0 \rightarrow t = \frac{7 \pm \sqrt{49}}{2}$

$a^2 = \frac{7+\sqrt{49}}{2} \rightarrow \begin{cases} a_1 = \sqrt{\frac{7+\sqrt{49}}{2}} \\ a_2 = -\sqrt{\frac{7+\sqrt{49}}{2}} \end{cases} \rightarrow S = 0 \;,\; P = \left(-\frac{7+\sqrt{49}}{2}\right)$

$2p^2 - 12sp + 2s = 8q + 7\sqrt{49}$

$2x^2 - ax + b = 0 \qquad \alpha+\beta = +\frac{a}{2} \;,\; \alpha\cdot\beta = \frac{b}{2}$

$2ax^2 + ax - 4 = 0 \qquad \alpha'+\beta' = -\frac{a}{2} \;,\; \alpha'\cdot\beta' = -\frac{4}{2} = -2$

$(\alpha - \frac{1}{2}) + (\beta - \frac{1}{2}) = \alpha + \beta - 1$

$(\alpha - \frac{1}{2})(\beta - \frac{1}{2}) = -2 \rightarrow \alpha\beta - \frac{1}{2}\alpha - \frac{1}{2}\beta + \frac{1}{4} = \alpha\beta - \frac{1}{2}(\alpha+\beta) + \frac{1}{4} \Rightarrow$

$-2 = \frac{b}{2} - \frac{1}{2}\left(\frac{a}{2}\right) + \frac{1}{4} \rightarrow -\frac{8}{4} - \frac{1}{4} = \frac{b}{2} - \frac{a}{4} = \frac{2b-a}{4} \Rightarrow 2b - a = -15$

$\alpha + \beta - 1 = -\frac{a}{2} \rightarrow \frac{a}{2} = 1 \rightarrow a = 1 \Rightarrow 2b - 1 = -15 \rightarrow 2b = -14 \rightarrow b = -7$

$\left[\frac{ab}{4}\right] = \left[\frac{1\times(-7)}{4}\right] = \left[\frac{-7}{2}\right] = [-3.5] = -4$

$ax^2 - ax - b = 0 \rightarrow \alpha+\beta = 1 \;,\; \alpha\cdot\beta = -\frac{b}{a}$

$\begin{cases} 40\beta^2 + 20a^2 - 20\beta = 17 \\ a = 40 \rightarrow 40\beta^2 - 40\beta - b = 0 \\ a = 20 \rightarrow 20a^2 - 20a - b = 0 \end{cases}$ (+)

$40\beta^2 + 20a^2 - 40\beta - 20a - 2b = 0 \rightarrow 17 - 20\beta - 20\alpha - 2b = 0$

$40\beta^2 + 20a^2 - 20\beta = 17$

$17 - 20(\alpha+\beta) - 2b = 0$

$17 - 20 = 2b \rightarrow 2b = -3$

$b = -\frac{3}{2}$

$\alpha = 1 - \beta \qquad 4\beta^2 + 2(1-\beta)^2 - 2\beta = 1 \Rightarrow 2\beta(\beta - 1) = -1 \rightarrow \alpha\beta = \frac{1}{2} = \frac{-b}{a}$

$\Rightarrow a = -2b \qquad ax^2 - ax - b = 0 \rightarrow -2bx^2 + 2bx - b = 0 \qquad |\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{2}{\sqrt{5}}$

$\div 2 \;(10, 9, 8$