۱)

$3x^2 - ax + 4 = 0 \qquad x_1 = 3x_2 \Rightarrow x_1 \cdot x_2 = \frac{c}{a} \Rightarrow$

$3x_2 \cdot x_2 = \frac{4}{3} \to x_2^2 = \frac{4}{9} \to x_2 = \pm\frac{2}{3}$

$x_2 = \frac{2}{3} \to x_1 = 3 \times \frac{2}{3} = 2 \to x_1 + x_2 = \frac{8}{3} \to \frac{8}{3} = \frac{a}{3} \to \boxed{a = 8}$

$x_2 = \frac{-2}{3} \to x_1 = 3 \times \frac{-2}{3} = -2 \to x_1 + x_2 = -\frac{8}{3} \to -\frac{8}{\cancel{3}} = \frac{a}{\cancel{3}} \to \boxed{a = -8}$

$\to 8 - (-8) = 16$

۲)

$36x^2 - (m+14)x + 1 = 0 \qquad \left(\sqrt{\frac{1}{x_1}} + \sqrt{\frac{1}{x_2}}\right)^2 = (5)^2$ توان ۲

$\frac{1}{x_1} + \frac{1}{x_2} + 2\sqrt{\frac{1}{x_1 x_2}} = 25 \to \frac{x_2 + x_1}{x_1 x_2} + 2\sqrt{\frac{1}{x_1 x_2}} = 25 \to$

$x_1 \cdot x_2 = \frac{c}{a} = \frac{1}{36} \Rightarrow 36(x_1 + x_2) + 2\sqrt{36} = 25 \to x_1 + x_2 = \frac{13}{36}$

$x_1 + x_2 = \frac{m+14}{\cancel{36}} = \frac{13}{\cancel{36}} \to m + 14 = 13 \to m = -1$

$mx^2 + 3x + 2 = 0 \xrightarrow{m=-1} -x^2 + 3x + 2 = 0 \to \alpha \cdot \beta = -2$

۳)

$4x^3 + kx^2 - 9x - 2 = 0 \quad , \alpha + \beta = 1 \quad , \alpha \cdot \beta = -2 \qquad k = ?$

۴)

$x^2 + 6x + a = 0 \qquad \alpha < \beta < 0$

$3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85 \qquad a = ?$

$\alpha + \beta = -6 \; , \; \alpha \cdot \beta = a \qquad \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\underbrace{2\alpha^2 + 2\beta^2} + \alpha^2 = 12\sqrt{2} + 85 \to 2(\alpha^2 + \beta^2) + \alpha^2 = 2(36 - 2a) + \alpha^2 = 72 - 4a + \alpha^2$

$\to \alpha^2 - 4a + 72 = 12\sqrt{2} + 85$

۱۵

$x^4 - 7x^2 - 5 = 0 \qquad x^2 = t$

$t^2 - 7t - 5 = 0 \rightarrow S = 7 \ , \ P = -5$

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$2x^2 - ax + b = 0 \qquad \alpha + \beta = +\frac{a}{2} \ , \ \alpha . \beta = \frac{b}{2}$

$2ax^2 + ax - 6 = 0 \qquad \alpha' + \beta' = -\frac{a}{2} \ , \ \alpha' . \beta' = -\frac{6}{2} = -3$

$(\alpha - \frac{1}{2}) + (\beta - \frac{1}{2}) = \alpha + \beta - 1$

$(\alpha - \frac{1}{2})(\beta - \frac{1}{2}) = -3 \rightarrow \alpha . \beta - \frac{1}{2}\alpha - \frac{1}{2}\beta + \frac{1}{4} = \alpha . \beta - \frac{1}{2}(\alpha + \beta) + \frac{1}{4} \Rightarrow$

$-3 = \frac{b}{2} - \frac{1}{2}(\frac{a}{2}) + \frac{1}{4} \rightarrow \cancel{-3 - \frac{1}{4}} \ \frac{-13}{\cancel{4}} = \frac{b}{2} - \frac{a}{4} = \frac{2b - a}{\cancel{4}} \Rightarrow 2b - a = -13$

$\underbrace{\alpha + \beta}_{\frac{a}{2}} - 1 = -\frac{a}{2} \rightarrow \cancel{2}\frac{a}{\cancel{2}} = 1 \rightarrow a = 1 \quad \Rightarrow 2b - 1 = -13 \rightarrow 2b = -12 \rightarrow b = -6$

$[\frac{ab}{4}] = [\frac{1 \times (-6)}{4}] = [\frac{-3}{2}] = [-1.5] = -2$

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$ax^2 - ax - b = 0 \rightarrow \alpha + \beta = 1 \ , \ \alpha . \beta = \frac{-b}{a}$

$40 . \beta^2 + 20 . \alpha^2 - 20 . \beta = 17$

$a = 40 \rightarrow \begin{cases} 40 . \beta^2 - 40 . \beta - b = 0 \\ 20 . \alpha^2 - 20 . \alpha - b = 0 \end{cases}$

$a = 20 \quad +$

$\cancel{40 . \beta^2 + 20 . \alpha^2 - 40 . \beta} - 20 . \alpha - 2b = 0 \rightarrow 17 - 20 . \beta - 20 . \alpha - 2b = 0$

$40 . \beta^2 + 20 . \alpha^2 - 20 . \beta = 17$

$17 - 20 . \underbrace{(\alpha + \beta)}_{1} - 2b = 0$

$17 - 20 = 2b \rightarrow 2b = -3$

$b = -3/2$