

پیرا امینی

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$$3u^2 - au + E = 0$$

$$u_1 = \alpha, u_2 = 3\alpha \rightarrow S = \alpha + 3\alpha = 4\alpha, P = \alpha \times 3\alpha = 3\alpha^2$$

$$0 = 5u^2 - Su + P \rightarrow u^2 - 4\alpha u + 3\alpha^2 = 0 \quad \left. \begin{array}{l} \times 3 \\ \end{array} \right\} \begin{array}{l} 3u^2 - 12\alpha u + 9\alpha^2 = 0 \\ 3u^2 - au + E = 0 \end{array}$$

$$\underline{-a = -12\alpha}, \quad 9\alpha^2 = E \rightarrow \alpha^2 = \frac{E}{9} \rightarrow \alpha = \pm \frac{\sqrt{E}}{3}$$

$$a = 12\alpha \rightarrow a_1 = 12 \times \frac{\sqrt{E}}{3} = 4\sqrt{E}, \quad a_2 = 12 \times \left(-\frac{\sqrt{E}}{3}\right) = -4\sqrt{E}$$

$$|a_1 - a_2| = |4\sqrt{E} - (-4\sqrt{E})| = |8\sqrt{E}| = \boxed{8\sqrt{E}}$$

۲-

$$34u^2 - (m+14)u + 1 = 0 \quad P = \frac{c}{a} = \frac{1}{34} \quad S = \frac{-b}{a} = \frac{m+14}{34}$$

$$\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 2\omega \quad \xrightarrow{\text{مربع}} \quad \frac{1}{\alpha} + \frac{1}{\beta} + 2\sqrt{\frac{1}{\alpha} \times \frac{1}{\beta}} = 4\omega^2 \rightarrow$$

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$$\frac{\beta + \alpha}{\alpha\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 4\omega^2 \rightarrow \frac{S}{P} + 2\sqrt{\frac{1}{P}} = 4\omega^2 \rightarrow \frac{\frac{m+14}{34}}{\frac{1}{34}} + 2\sqrt{\frac{1}{\frac{1}{34}}} = 4\omega^2$$

$$\frac{34(m+14)}{34} + \frac{2 \times \sqrt{34}}{2 \times 4} = 4\omega^2 \rightarrow m+14 + \frac{12}{4} = 4\omega^2 \rightarrow m = -1$$

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$$mu^2 + 3u + 1 = 0 \xrightarrow{m=-1} -u^2 + 3u + 1 = 0$$

$$\text{حاصل جمع ریشه ها} = \frac{c}{a} = \frac{1}{-1} = \boxed{-1}$$

Arman

$$Ea^3 + ka^2 - 9a - 2 = 0$$

$$\alpha + \beta = 1$$

$$\alpha\beta = -2$$

ریشه نامعلوم $\rightarrow$

$$E(a - \alpha)(a - \beta)(a - \gamma)$$

$$a^3 - (\alpha + \beta)a^2 + \alpha\beta$$

$$E(a^3 - (\alpha + \beta)a^2 + \alpha\beta)(a - \gamma) \rightarrow E[a^4 - (\alpha + \beta + \gamma)a^3 + (\alpha\beta + \gamma(\alpha + \beta))a^2 - \alpha\beta\gamma] = 0$$

$$Ea^4 - E(\alpha + \beta + \gamma)a^3 + E(\alpha\beta + \gamma(\alpha + \beta))a^2 - E\alpha\beta\gamma = 0$$

$$\begin{cases} -E(\alpha + \beta + \gamma) = k \\ E(\alpha\beta + \gamma(\alpha + \beta)) = -9 \\ -E\alpha\beta\gamma = -2 \end{cases} \quad \begin{matrix} \alpha\beta = -2 \\ \alpha + \beta = 1 \end{matrix} \quad \begin{cases} -E(1 + \gamma) = k \\ E(-2 + \gamma(1)) = -9 \\ -E(-2)\gamma = -2 \end{cases}$$

$$\textcircled{1} E(-2 + \gamma) = -9 \rightarrow -2 + E\gamma = -9 \rightarrow E\gamma = -7 \rightarrow \gamma = -\frac{7}{E}$$

$$\textcircled{2} 2\gamma = -2 \Rightarrow \gamma = -\frac{1}{E}$$

$$k = -E(1 + \gamma) = -E\left(1 - \frac{1}{E}\right) = -E\left(\frac{E-1}{E}\right) = \boxed{-3}$$

$$x^2 + 4x + a = 0$$

$$\alpha < \beta < 0$$

$$3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 18$$

$$\alpha + \beta = \frac{-b}{a} = -4 \rightarrow \alpha + \beta = -4 \rightarrow \alpha = -4 - \beta$$

$$3(-4 - \beta)^2 + 2\beta^2 = 12\sqrt{2} + 18 \rightarrow 3(16 + \beta^2 + 12\beta) + 2\beta^2 =$$

$$12\sqrt{2} + 18 \rightarrow 10\beta + 3\beta^2 + 48 + 2\beta^2 = 12\sqrt{2} + 18 \rightarrow$$

$$5\beta^2 + 10\beta + 10 - 12\sqrt{2} = 0 \rightarrow 5\beta^2 + 10\beta + (10 - 12\sqrt{2}) = 0$$

$$\alpha = -3 - 2\sqrt{2} \quad \beta = -3 + 2\sqrt{2}$$

$$10 \quad \alpha^2 = (-3 - 2\sqrt{2})^2 = 9 + 4\sqrt{2} + 2\sqrt{2}$$

$$\beta^2 = (-3 + 2\sqrt{2})^2 = 9 - 4\sqrt{2} + 2\sqrt{2}$$

$$3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 18 \rightarrow 3(9 + 4\sqrt{2} + 2\sqrt{2}) + 2(9 - 4\sqrt{2} + 2\sqrt{2})$$

$$= 12\sqrt{2} + 18 \rightarrow 27 + 18\sqrt{2} + 4\sqrt{2} + 18 - 12\sqrt{2} + 8\sqrt{2} =$$

$$12\sqrt{2} + 18$$

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$$\left\{ \begin{array}{l} \text{عبارت} \rightarrow 27 + 18 = 45 \\ \sqrt{2} \text{ جمله} \rightarrow 18\sqrt{2} - 12\sqrt{2} = 4\sqrt{2} \\ 2\sqrt{2} \text{ جمله} \rightarrow 4\sqrt{2} + 8\sqrt{2} = 12\sqrt{2} \end{array} \right.$$

$$(45 + 12\sqrt{2}) + (4\sqrt{2}) = 49 + 16\sqrt{2}$$

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$$49 = 12 \rightarrow 9 = 2$$

$$45 + 12\sqrt{2} = 49 \rightarrow 12\sqrt{2} = 4 \rightarrow \sqrt{2} = \frac{1}{3} \rightarrow 2 = \pm 2 \rightarrow \left. \begin{array}{l} \\ \end{array} \right\} 2 = 2$$

Arman

ادامہ کم۔

$$\begin{aligned}
 \alpha < \beta < 0 &\rightarrow -3 - 2\sqrt{2} < -3 + 2\sqrt{2} < 0 \\
 -2\sqrt{2} < 2\sqrt{2} &\rightarrow 4\sqrt{2} > 0 \rightarrow 2 > 0 \\
 -3 + 2\sqrt{2} < 0 &\rightarrow 2\sqrt{2} < 3 \Rightarrow 2 < \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2} \approx 2.12
 \end{aligned}
 \left. \vphantom{\begin{aligned} \alpha < \beta < 0 \\ -2\sqrt{2} < 2\sqrt{2} \\ -3 + 2\sqrt{2} < 0 \end{aligned}} \right\} 0 < 2 < 2.12$$

در رابط قبلی $2 = 2$ و در شرط $0 < 2 < 2.12$ صدق می کند. قابل قبول

$$\begin{aligned}
 \frac{c}{a} = a = \alpha\beta &\rightarrow a = (-3 - 2\sqrt{2})(-3 + 2\sqrt{2}) = (-3)^2 - (2\sqrt{2})^2 \\
 9 - (2 \times 2) &\xrightarrow{2=2} 9 - (4 \times 2) = 9 - 8 = 1
 \end{aligned}$$

$$10 \quad a = 1 \rightarrow 2x^2 + 4x + 1 = 0$$

$$\text{ریشه} = \frac{-4 \pm \sqrt{16 - 8}}{2} = \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2}$$

$$-3 \pm 2\sqrt{2}$$

$$\alpha = -3 - 2\sqrt{2}$$

$$15 \quad \beta = -3 + 2\sqrt{2}$$

پہلے اسے

$$u^2 - v u^2 - a = 0$$

$$v P^2 - 3SP + 2S = ? \quad - \omega$$

$$u^2 = t \rightarrow t^2 - vt - a = 0$$

$$t = \frac{v \pm \sqrt{49}}{2} \rightarrow \frac{v - \sqrt{49}}{2} < 0 \quad \text{عق}$$

$$\Delta = 49 - 4(1)(-a) = 49$$

$$t = u^2 > 0$$

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$$t = \frac{v + \sqrt{49}}{2} \quad u = \pm \sqrt{\frac{v + \sqrt{49}}{2}} \rightarrow P = \sqrt{\frac{v + \sqrt{49}}{2}} \times \left(-\sqrt{\frac{v + \sqrt{49}}{2}} \right)$$

$$= \frac{-v - \sqrt{49}}{2}$$

$$S = \sqrt{\frac{v + \sqrt{49}}{2}} - \sqrt{\frac{v + \sqrt{49}}{2}} = 0$$

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$$3SP = 0, \quad 2S = 0 \rightarrow 2P^2 = 2 \left(\frac{-v - \sqrt{49}}{2} \right)^2 = \frac{49 + 49 + 14\sqrt{49}}{2}$$

$$= \frac{49 + 14\sqrt{49}}{2} = \boxed{49 + 7\sqrt{49}}$$

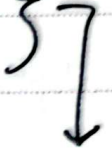
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$$d: 2m^2 - am + b = 0 \quad \alpha + 0, \omega, \beta + 0, \omega$$

$$d': 2an^2 + an - 4 = 0 \quad \alpha, \beta$$

$$d: S = \alpha + \beta + 0, \omega + 0, \omega = \alpha + \beta + 1 \rightarrow S = -\frac{b}{a} = \frac{a}{2} = \alpha + \beta + 1$$

$$d': S = \alpha + \beta = -\frac{b}{a} = \frac{-a}{2a} = -\frac{1}{2}$$



$$\alpha + \beta = -\frac{1}{2} \rightarrow \alpha + \beta + 1 = -\frac{1}{2} + 1 = \frac{1}{2} \rightarrow \frac{a}{2} = \frac{1}{2} \rightarrow a = 1$$

$$d: 2m^2 - am + b = 0 \rightarrow (\alpha + 0, \omega)(\beta + 0, \omega) = \alpha\beta + \frac{1}{2}\alpha + \frac{1}{2}\beta + \frac{1}{2}$$

$$d': 2an^2 + an - 4 = 0 \rightarrow \alpha\beta = \frac{c}{a} = \frac{-4}{2} = -2$$



$$d: P = \alpha\beta + \frac{1}{2}(\alpha + \beta) + \frac{1}{2} \rightarrow -2 - \frac{1}{2} + \frac{1}{2} = -\frac{12}{2} + \frac{1}{2} = -\frac{11}{2}$$

$$d: P = \frac{c}{a} = \frac{b}{2} = -\frac{11}{2} \rightarrow \frac{2b}{2} = -\frac{11}{2} \rightarrow 2b = -11 \rightarrow b = -\frac{11}{2}$$

$$15 \left[\frac{ab}{2} \right] = \left[\frac{1 \times -\frac{11}{2}}{2} \right] = \left[-\frac{11}{4} \right] = \left[\frac{-11}{4} \right] = [-1, 4, 2] = -2$$

$$ax^2 + bx + c = 0$$

$$x_1 \cdot x_2 + x_1 \cdot x_2 - x_1 \cdot x_2 = 1 \rightarrow x_1^2 + x_2^2 - x_1 x_2 = \frac{1}{x_1}$$

$$x_1 + x_2 = \frac{-(c-a)}{a} = \frac{a}{a} = 1 \rightarrow x_1 = 1 - x_2$$

$$x_1 x_2 = \frac{-(c+b)}{a} = \frac{b}{a}$$

$$x_1^2 + (1-x_1)^2 - x_1 = \frac{1}{x_1} \rightarrow x_1^2 + 1 + x_1^2 - 2x_1 - x_1 = \frac{1}{x_1}$$

$$2x_1^2 - 3x_1 + 1 = \frac{1}{x_1} \rightarrow 2x_1^3 - 3x_1^2 + x_1 - \frac{1}{x_1} = 0 \rightarrow$$

$$2x_1^3 - 3x_1^2 + x_1 - \frac{1}{x_1} = 0 \rightarrow 2x_1^4 - 3x_1^3 + x_1^2 - 1 = 0 \rightarrow$$

از آنجایی که ریشه های این معادله همان ریشه های معادله اصلی است پس حاصل ضرب ریشه های این معادله همان حاصل ضرب ریشه های معادله اصلی است.

$$(x_1 - x_2)^2 = \frac{x_1^2 + x_2^2 - 2x_1 x_2}{(x_1 + x_2)^2 - 4x_1 x_2} \rightarrow (x_1 + x_2)^2 - 4x_1 x_2$$

$$15 \rightarrow x_1 x_2 = \frac{c}{a} = \frac{1}{x_1} \rightarrow (x_1 - x_2)^2 = \frac{(x_1 + x_2)^2 - 4x_1 x_2}{(x_1 + x_2)^2 - 4x_1 x_2} = \frac{1 - \frac{4}{x_1}}{1 - \frac{4}{x_1}} \rightarrow$$

$$1 - \frac{4}{x_1} = 1 - \frac{4}{\omega} = \frac{\omega - 4}{\omega} \rightarrow (x_1 - x_2)^2 = \frac{\omega - 4}{\omega} \rightarrow (x_1 - x_2) = \frac{\sqrt{\omega - 4}}{\sqrt{\omega}}$$

$$\frac{\sqrt{\omega - 4}}{\sqrt{\omega}}$$

$$x^2 - (a+1)x + a = 0 \quad \alpha, \alpha+1 \quad -1$$

$$P = \frac{c}{a} = \frac{a}{1} = \alpha(\alpha+1) \Rightarrow \alpha^2 + \alpha = a$$

$$S = -\frac{b}{a} = \frac{a+1}{1} = \alpha + \alpha + 1 = 2\alpha + 1 = a+1 \rightarrow a = 2\alpha + 1$$

$$2\alpha + 1 = \alpha^2 + \alpha \rightarrow \alpha^2 - \alpha = 0 \rightarrow \alpha = \pm 1 \quad \text{طبیعی} \rightarrow \alpha = 1, a = 3$$

$$x^2 - (2a+1)x + b = 0 \quad \beta, \beta+1$$

$$S = \beta + \beta + 1 = 2\beta + 1 = -\frac{b}{a} = \frac{2a+1}{1} \rightarrow 2\beta + 1 = 2a + 1 \rightarrow 2\beta = 2a \rightarrow \beta = a$$

$$P = \beta(\beta+1) = \beta^2 + \beta = \frac{c}{a} = b \rightarrow (\beta)^2 + \beta = b$$

$$|P_1 - P_2| = |a - b| = |3 - 2\beta| = 2$$

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-9

$$x^2 + x - 1 - m^2 = 0$$

$$S = -\frac{b}{a} = -\frac{1}{1} = -1$$

$$P = \frac{1-m^2}{1} = -(1+m^2)$$

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$$\alpha^2 + \beta^2 = S^2 - 2P = (-1)^2 - 2(-(1+m^2)) = 1 + 2 + 2m^2 = 2m^2 + 3$$

$$x_{\min} = x_s = -\frac{b}{2a} = \frac{-0}{2} = 0$$

$$y_{\min} = y_s = 2(0)^2 + 3 = 3$$

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بررسی وجود ریشه حاد و حاد:

$$\Delta = b^2 - 4ac = (1)^2 - 4(1)(-(1+m^2)) \geq 0 \rightarrow 1 + 4(1+m^2) \geq 0$$

$$1 + 4 + 4m^2 \geq 0 \rightarrow 5 + 4m^2 \geq 0 \quad \text{همواره برقرار} \quad \checkmark$$

Arman

۱۰

عرفان بیان $B(-1, y), A(3, y) \rightarrow$ محد، تقارن سیمی $\leftarrow$ وسط AB

$$h = \frac{3-(-1)}{2} = \frac{2}{2} = 1$$

اس، $5 \mid -1$

$$y = a(h-h)^2 + 1 \rightarrow y = a(h - (-1))^2 + 1 \rightarrow$$

$$y = a(h+1)^2 + 1 \xrightarrow{A(3, y)} y = a(3+1)^2 + 1 \rightarrow$$

$$y = a(4)^2 + 1 \rightarrow y = 16a + 1$$

$$10 \text{ ریشہ} \rightarrow 0 = a(h+1)^2 + 1 \rightarrow a(h+1)^2 = -1 \rightarrow$$

$$(h+1)^2 = -\frac{1}{a} \rightarrow a(h^2 + 1 + 2h) + 1 = 0$$

سیمی محدود، تقاطع می کند و $(-1, 1) \rightarrow$ (اس سیمی)پس دستانه رو به پایین $\leftarrow a < 0$

$$15 \rightarrow ah^2 + 2ah + (a+1) = 0$$

$$\alpha + \beta = -\frac{2a}{a} = -2$$

$$\alpha\beta = \frac{a+1}{a} = 1 + \frac{1}{a}$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (-2)^2 - 2(1 + \frac{1}{a}) = 0 \rightarrow$$

$$4 - 2 - \frac{2}{a} = 0 \rightarrow -\frac{2}{a} = -2 \rightarrow -2a = 2 \rightarrow a = -\frac{2}{2} = -1$$

۲۰ با شرت $a < 0$ سازگار است.

$$y = -\frac{2}{3}(h+1)^2 + 1 \xrightarrow{h=0} -\frac{2}{3}(0+1)^2 + 1 = -\frac{2}{3} + 1 = \frac{1}{3}$$

سیمی محدود، تقاطع می کند

Arman $(0, \frac{1}{3})$