سوال ۱)

$2x^2+y^2-4x+6y+k=0 \to \underbrace{2x^2-4x+\boxed{2}}_{2(x^2-2x+1)=2(x-1)^2}+\underbrace{y^2+6y+\boxed{9}}_{(y+3)^2}=0$

$k=2+9=11$

سوال ۲)

$f(x)=\begin{cases} x^2+4x & ; x\geq a \\ 2x+a+2 & ; x<a \end{cases}$

$a^2+4a=2a+a+2$

$\to a^2+a-2=0 \to (a+2)(a-1)=0$

$\hookrightarrow a=-2, 1 \; ; a>0$

$\Rightarrow f(1) \rightsquigarrow x^2+4x \to 1+4(1)=5$

سوال ۳) $a+b=?$

$f(x)=\begin{cases} 2x^2-b & ; |x+1|\geq 2 \\ a+2x & ; -3\leq x\leq -1 \end{cases}$

$\to x+1\geq 2 \to x\geq 1$

$\hookrightarrow x+1\leq -2 \to x\leq -3$

$\overset{x=-3}{\rightsquigarrow} \quad 18-b=a-6 \to a+b=24$

سوال ۴) اگر دامنهٔ $y=\sqrt{4x-a}+\sqrt{b-3x}$ برابر $\{2\}$ باشد، $\frac{b}{a}=?$

$4x-a\geq 0 \to a\leq 4x \to x\geq \frac{a}{4} \Rightarrow \frac{a}{4}=2 \Rightarrow a=12$

$b-3x\geq 0 \to 3x\leq b \to x\leq \frac{b}{3} \Rightarrow \frac{b}{3}=2 \Rightarrow b=6$

$\frac{6}{12}, \frac{1}{2}$

سوال ۶ با سوال ۵ جابه‌جا نوشتم.

سوال ۶)

الف) $y=\sqrt{\frac{x^2-x-6}{x^4-13x^2+36}}=\sqrt{\frac{(x-3)(x+2)}{(x^2-4)(x^2-9)}}, \quad \frac{(x-3)(x+2)}{(x+2)(x-2)(x+3)(x-3)}\geq 0$

$\to x=3^*, -2^*, 2, -3$

$-3 \quad -2^* \quad 2 \quad 3^*$

$+ \quad - \quad - \quad + \quad +$

$\to D_f:(-\infty,-3)\cup(2,3)\cup(3,+\infty)$

ب) $y=\sqrt{\frac{x^3-2x^2-5x+6}{x^3+2x^2-5x-6}}, \quad \frac{(x-1)(x^2-x-6)}{(x+1)(x^2+x-6)}, \quad \frac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)}\geq 0$

$x=1$ ریشه

$x=-1$ ریشه

$x=1, 3, -2, -1, -3, 2$

$-3 \quad -2 \quad -1 \quad 1 \quad 2 \quad 3$

$+ \quad - \quad + \quad - \quad + \quad - \quad +$

$\to D_f:(-\infty,-3)\cup[-2,-1)\cup[1,2)\cup[3,+\infty)$

سوال ۱)

$2x^2+y^2-4x+6y+k=0 \to \underbrace{2x^2-4x+2}_{2(x^2-2x+1)=2(x-1)^2}+\underbrace{y^2+6y+9}_{(y+3)^2}=0$

$k=2+9=11$

سوال ۲)

$f(x)=\begin{cases} x^2+4x & ; x\geq a \\ 2x+a+2 & ; x<a \end{cases}$

$a^2+4a=2a+a+2$

$\to a^2+a-2=0 \to (a+2)(a-1)=0$

$\hookrightarrow a=-2, 1 \; ; \; a>0$

$\to f(1) \to x^2+4x \to 1+4(1)=5$

سوال ۳) $a+b=?$

$f(x)=\begin{cases} 2x^2-b & ; |x+1|\geq 2 \\ a+2x & ; -3<x<-1 \end{cases}$

$|x+1|\geq 2 \to x+1\geq 2 \to x\geq 1$

$\hookrightarrow x+1\leq -2 \quad x\leq -3$

$\xrightarrow{x=-3} 18-b=a-6 \to a+b=24$

سوال ۴) اگر دامنه ی $\sqrt{6x-a}+\sqrt{b-3x}$ برابر $\{2\}$ باشد، $\frac{b}{a}=?$

$6x-a\geq 0 \to a\leq 6x \to x\geq \frac{a}{6} \Rightarrow \frac{a}{6}=2 \Rightarrow a=12$

$b-3x\geq 0 \to 3x\leq b \to x\leq \frac{b}{3} \Rightarrow \frac{b}{3}=2 \Rightarrow b=6$

$\frac{6}{12}=\frac{1}{2}$

با سوال ۵ جا به جا نوشتم.

سوال ۶)

الف) $y=\sqrt{\frac{x^2-x-6}{x^4-13x^2+36}}=\sqrt{\frac{(x-3)(x+2)}{(x^2-4)(x^2-9)}} \quad , \quad \frac{(x-3)(x+2)}{(x+2)(x-2)(x+3)(x-3)}\geq 0$

$\to x=3^*, -2^*, 2, -3$

$-3 \quad -2^* \quad 2 \quad 3^*$

$+ \quad - \quad - \quad + \quad +$

$\to D_f=(-\infty,-3)\cup(2,3)\cup(3,+\infty)$

ب) $y=\sqrt{\frac{x^3-2x^2-5x+6}{x^3+2x^2-5x-6}} \quad , \quad \frac{(x-1)(x^2-x-6)}{(x+1)(x^2+x-6)}=\frac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)}\geq 0$

$x=1$ ریشه

$x=-1$ ریشه

$\to x=1, 3, -2, -1, -3, 2$

$-3 \quad -2 \quad -1 \quad 1 \quad 2 \quad 3$

$+ \quad - \quad + \quad - \quad + \quad - \quad +$

$\to D_f=(-\infty,-3)\cup[-2,-1)\cup[1,2)\cup[3,+\infty)$

Subject: Year. Month. Day.

$\frac{x+1}{|x-3|}\geq 0 \to x=-1,3$

$\frac{4-x}{x^2+2x+4}\geq 0 \to x\leq 4$ همیشه مثبت

سوال ۶)

الف) $y=\sqrt{\frac{x+1}{|x-3|}}+\sqrt{\frac{4-x}{x^2+2x+4}}$

$-1 \quad 3$: $- \ \circ \ + \ \circ \ +$ $\to [-1,3)\cup(3,+\infty)$ ①

4 : $+ \ \circ \ -$ $\to (-\infty,4]$ ②

$-2 \ -1 \ 0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6$

$\to$ ①$\cap$② $\to D_f=[-1,4]-\{3\}$

ب) $y=\sqrt{[x]-4}+\sqrt{2-[x]} \to 2-[x]\geq 0 \to [x]\leq 2 \to (-\infty,3)$ ②

$[x]-4\geq 0 \quad [x]\geq 4 \to x\in[4,+\infty)$ ①

①$\cap$② $\to D_f=\varnothing$

سوال ۷)

ز) $y=\sqrt{[-x]-2} \to [-x]-2\geq 0 \quad [-x]\geq 2 \to D_f=(-\infty,-3)$

$(-\infty,-2]$

۱,۷۵

ح) $y=\frac{5x^2+4}{[x]^2-2[x]-3} \to [x]^2-2[x]-3>0 \to ([x]-3)([x]+1)>0$

$[x]-3=0 \to [x]=3 \to x\in[3,4) \ , \ [x]+1=0 \to [x]=-1 \to x\in[-1,0)$

$-1 \quad 0 \quad 3 \quad 4$: $+ \ \circ \ \circ \ + \ \circ \ \circ \ +$ $\to D_f=R-[1,0)\cup\{3,4)$

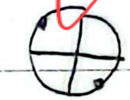
سوال ۸)

$y=\frac{\cot x+1}{\tan x+1}=\frac{\frac{\cos x}{\sin x}+1}{\frac{\sin x}{\cos x}+1} \to \sin x\neq 0 \to$

$\frac{\sin x}{\cos x}\neq -1 \to$

$D_f: R-\{k\pi, k\pi-\frac{\pi}{4}\}$

$y=\dfrac{\cot x+1}{\tan x+1}=\dfrac{\frac{\cos x}{\sin x}+1}{\frac{\sin x}{\cos x}+1}$ $\longrightarrow \sin x\neq 0 \rightarrow$ 

$\frac{\sin x}{\cos x}+1 \rightarrow \frac{\sin x}{\cos x}\neq -1 \rightarrow$ 

$\hookrightarrow \cos x\neq 0 \rightarrow$ 

$R-$ $D_f=\{\frac{k\pi}{2}, k\pi-\frac{\pi}{4}\}$

سوال ۱۸

$D_f=[k\pi-\frac{\pi}{6}, k\pi+\frac{\pi}{6}]$

$y=\sqrt{1-4\sin^2 x} \rightarrow 1-4\sin^2 x\geq 0 \rightarrow 4\sin^2 x\leq 1 \rightarrow \sin^2 x\leq \frac{1}{4} \rightarrow -\frac{1}{2}\leq \sin x\leq \frac{1}{2}$ 

$\rightarrow x-1>0 \rightarrow x>1$

سوال ۱۹ الف) $y=\sqrt{1-\log_{\frac{1}{2}}^{x-1}} \rightarrow 1-\log_{\frac{1}{2}}^{x-1}\geq 0 \quad \log_{\frac{1}{2}}^{x-1}\leq 1 \rightarrow x-1\geq \frac{1}{2} \rightarrow x\geq \frac{3}{2}$

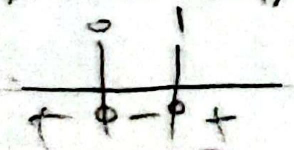
$\rightarrow D_f=[\frac{3}{2},+\infty)$

ب) $y=\dfrac{\sqrt{x^2-x}}{1-\log_4^{x^2-3x}} \rightarrow 1-\log_4^{x^2-3x}\neq 0 \quad \log_4^{x^2-3x}\neq 1 \rightarrow x(x-3)\neq 4$

$x^2-3x-4\neq 0$

$(x-4)(x+1)\neq 0$

$\rightarrow x(x-1)\geq 0$

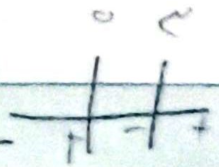


$\rightarrow x\in(-\infty,0]\cup[1,+\infty)$ (1)

$\rightarrow x\neq 4,-1$ (2)

$x^2-3x>0$

$x(x-3)>0$



$x\in(-\infty,0)\cup(3,+\infty)$ (3)

(1) ∩ (2) ∩ (3) $=(-\infty,0)\cup(3,+\infty)-\{4,-1\}$

سوال ۱۰) الف) $y=\sqrt{2^{4x-x^2}-8} \rightarrow 2^{4x-x^2}-2^3 \geq 0 \quad 2^{4x-x^2} \geq 2^3 \rightarrow -x^2+4x \geq 3$

$\rightarrow -x^2+4x-3 \geq 0 \quad \rightarrow -(x^2-4x+3) \geq 0 \quad -(x-3)(x-1) \geq 0$

	1		3	
$-$		$+$		$-$

$D_f=[1,3]$

۵

ب) $y=\left(\dfrac{3x+5}{2x+4}\right)! \qquad \dfrac{3x+5}{2x+4} \in W \rightarrow 3x+5 \in 2xW+4W$

$3x-2xW \in 4W-5$

$x(3-2W) \in 4W-5$

$x \in \dfrac{4W-5}{3-2W}$

$D_f=\left\{x \mid x=\dfrac{4k-5}{3-2k},\ k \in W\right\}$