

19/5

Year: 1401 Month: May Day: 2 ()

Subject: Algebra

$$x^2 + y^2 - 2x + 4y + K = 0$$

مربع کامل

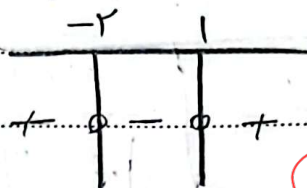
(1)

$$(y+2)^2 + (\sqrt{2}x - \sqrt{2})^2 = 0$$

$$K = 9 + 1 = 10 \rightarrow K = 10$$

(5)

$$f(x) = \begin{cases} x^2 + 2x & x \geq a \\ 2x + a + 1 & x < a \end{cases} \quad f(1) = 3$$



If $x \leq a \rightarrow a^2 + 2a = 2a + a + 1 \rightarrow a^2 + 2a = 2a + 1$

$$a^2 + a + 1 = 0 \rightarrow (a+1)(a-1) = 0$$

$$a = -1 \quad a = 1$$

$f(1) = (1)^2 + 2(1) = 3$ جواب

$$f(x) = \begin{cases} x^2 - b & |x+1| \geq 2 \\ a + 2x & -3 \leq x \leq -1 \end{cases} \quad a+b$$

$$x \leq -3 \rightarrow x^2 - b \leq a + 2x$$

$$1 - b \leq a - 4 \rightarrow 1 + 4 \leq a + b \leq 15$$

$$y = \sqrt{4x - a} + \sqrt{b - 3x} \quad \{r\} \rightarrow 0 \quad \frac{b}{a} \leq \frac{1}{4} \quad (r)$$

$$\begin{aligned} 4x - a > 0 &\rightarrow 4x > a \rightarrow x > \frac{a}{4} \\ b - 3x > 0 &\rightarrow b > 3x \rightarrow \frac{b}{3} > x \end{aligned} \left\{ \frac{b}{3}, x > \frac{a}{4} \right.$$

$$\frac{b}{3} \leq \frac{1}{4} \left\{ \begin{aligned} b &\leq 3 \\ a &\leq 12 \end{aligned} \right. \leftarrow \frac{b}{3} = \frac{a}{4} \leq 1 \leftarrow \frac{b}{a} = \frac{1}{4}$$

$$\text{ii)} y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+2x+2}} \rightarrow 2 - 2(1)(2) = -12$$

$$x+1 > 0 \rightarrow x > -1$$

$$[-1, +\infty)$$

$$\begin{array}{c} 4 \\ + \quad - \\ \hline \end{array}$$

$$(-\infty, 4]$$

$$D = [-1, 4] - \{3\}$$

$$\text{iii)} y = \sqrt{[x] - 2} + \sqrt{2 - [x]} \quad (-\infty, 3)$$

$$[r, \omega)$$

$$[r, 3)$$

$$\begin{array}{c} r \quad \omega \\ + \quad - \\ \hline \end{array}$$

$$[r, \omega) \cup (\omega, +\infty)$$

$$\begin{array}{c} r \quad 3 \\ + \quad - \\ \hline \end{array}$$

$$D \rightarrow \text{استراحت ندارد} \rightarrow D = \emptyset$$

الف) $y = \sqrt{x^2 - x - 6} \rightarrow (x-3)(x+2)$ (4)

$x^2 - 13x + 36 \rightarrow x^2 \pm 6$

$t^2 - 13t + 36 \rightarrow (t-4)(t-9) \rightarrow x^2 \pm 4, \pm 9$

$\downarrow 4 \quad \downarrow 9$

$(-\infty, -2) \cup (2, +\infty) - \{2\}$

← $\frac{-x \pm \sqrt{x^2 - 4}}{2}$

$\begin{array}{ccccccc} & -2 & -2 & 2 & 2 & & \\ & | & | & | & | & & \\ + & - & - & - & + & + & + \end{array}$

ب) $\sqrt{x^2 - 2x^2 - 5x + 4} \div (x-1) \rightarrow (x-1)(x-2)(x-2)$

$\sqrt{x^2 + 2x^2 - 5x - 4} \rightarrow (1) \quad (2) \quad (2)$

$(-2) \quad (-2) \quad (-1) \quad (1) \quad (2) \quad (2)$ $\rightarrow$ $\frac{-x \pm \sqrt{x^2 - 4}}{2}$ $\rightarrow$ $(-2, -1, 2)$

$+ \quad - \quad - \quad + \quad - \quad -$

$D = (-\infty, -2) \cup [-2, -1) \cup [1, 2) \cup [2, +\infty)$

ج) $y = \sqrt{[-x] - 2} \quad [x] \geq 2 \rightarrow (-\infty, -2]$ (5)

د) $y = \sqrt{[x]^2 - 2[x] - 3}$

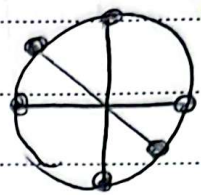
$([x]+1)([x]-3) \neq 0$

$\neq -1 \rightarrow [-1, 0)$

$\neq 3 \rightarrow \mathbb{R} - [3, 4)$

$D = \mathbb{R} - ([3, 4) \cup [-1, 0))$

الف) $\frac{\cot x + 1}{\tan x + 1} \rightarrow \tan x \neq -1$
 $\tan x + 1 \rightarrow$ صورت و مخرج نباید صفر شود

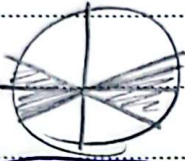


(1)

$$D_f = IR - \left\{ \frac{k\pi}{2}, k\pi - \frac{\pi}{2} \right\}$$

(5)

ب) $\sqrt{1 - \varepsilon \sin^2 x} \rightarrow 1 - \varepsilon \sin^2 x > 0 \rightarrow \sin^2 x < \frac{1}{\varepsilon}$
 $\frac{1}{\sqrt{\varepsilon}} < \sin x < \frac{1}{\sqrt{\varepsilon}} \rightarrow \cot x$
 $\left[k\pi - \frac{\pi}{6}, k\pi + \frac{\pi}{6} \right]$



10 الف) $\sqrt{1 - \log \frac{1}{x-1}} \rightarrow 1 - \log \frac{1}{x-1} \geq 0 \rightarrow \log \frac{1}{x-1} \leq 1$

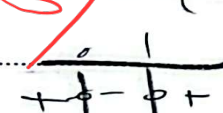
(9)

$x-1 > 0 \rightarrow x > 1$

$x \rightarrow \left[\frac{1}{10}, +\infty \right)$

15 $x-1 > \frac{1}{10} \rightarrow x > \frac{11}{10}$

ب) $\sqrt{x^2 - x} \rightarrow x(x-1) \geq 0$
 $1 - \log \frac{x^2 - 3x}{\varepsilon} \rightarrow \frac{x^2 - 3x}{1 - \log \varepsilon} \neq 0$



20 $\frac{x^2 - 3x}{1 - \log \varepsilon} \neq 0 \rightarrow \log \varepsilon \neq 1 \rightarrow x(x-1) > 0$

$(-\infty, 0) \cup (1, +\infty)$

$x^2 - 3x \neq \varepsilon \rightarrow x^2 - 3x - \varepsilon \neq 0$

$\rightarrow (x+1)(x-\varepsilon) \neq 0 \rightarrow$

$x \rightarrow (-\infty, 0) \cup (1, +\infty) - \{ \varepsilon, -1 \}$

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$$y \leq \sqrt{x^2 - 1} \quad \checkmark \quad x^2 - 1 \geq 0 \rightarrow$$

$$x^2 - 1 \geq 1$$

$$x^2 - 1 < 0 \leftarrow x^2 \geq 1$$

$$(x-1)(x-3)$$

$$\begin{array}{c} 1 \quad 3 \\ + \quad - \quad + \end{array}$$

$$D_f \subset [1, 3]$$

$$\rightarrow \left(\frac{3x + \omega}{2x + 1} \right)! \rightarrow \frac{3x + \omega}{2x + 1} \in W \quad \text{بجای نماند}$$

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$$W \subset \frac{\varepsilon W - \omega}{2W - 3} \rightarrow K \subset \frac{\omega - \varepsilon W}{2W - 3} \rightarrow$$

$$D_f = \left\{ x \mid x = \frac{\omega - \varepsilon K}{2K - 3}, K \in W \right\}$$

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