پرنیا دیاجی — کلاس یازدهم تک‌سلسله A

①

$$2x^2+y^2-4x+6y+k=0 \Rightarrow \begin{cases} 2x^2-4x=2(x^2-2x)=2\left((x-1)^2-1\right)=2(x-1)^2-2 \\ y^2+6y=(y^2+6y+9)-9=(y+3)^2-9 \end{cases}$$

$$\Rightarrow 2(x-1)^2-2+(y+3)^2-9+k=0 \Rightarrow 2(x-1)^2+(y+3)^2+(k-11)=0$$

$$\Rightarrow \underbrace{2(x-1)^2+(y+3)^2}+(k-11)=0 \Rightarrow k-11=0 \Rightarrow \boxed{k=11}$$

در نقطه $(1,-3)$ برابر صفر اند

---

②

$$f(x)=\begin{cases} x^2+4x\,; & x\geq a \\ 2x+a+2\,; & x\leq a \end{cases} \Rightarrow a^2+4a=2a+a+2 \Rightarrow a^2+4a-3a-2=0 \quad (a>0)$$

$$\Rightarrow a^2+a-2=0 \Rightarrow (a+2)(a-1)=0 \Rightarrow \begin{cases} a=1 \ ✓ \\ a=-2 \ ✗ \end{cases}$$

$$f(1)=(1)^2+4(1)=\boxed{5}$$

$$2(1)+1+2=\boxed{5}$$

---

③

$$f(x)=\begin{cases} 2x^2-b\,; & |x+1|\geq 2 \\ a+2x\,; & -3\leq x\leq -1 \end{cases} \Rightarrow \begin{cases} x+1\geq 2 \Rightarrow x\geq 1 \\ x+1\leq -2 \Rightarrow x\leq -3 \end{cases}$$

$a+b=?$

$$\Rightarrow 2(-3)^2-b=a+2(-3) \Rightarrow 18-b=a-6 \Rightarrow \boxed{a+b=24}$$

---

④

$$y=\sqrt{6x-a}+\sqrt{b-3x} \Rightarrow \begin{cases} 6x-a\geq 0 \Rightarrow 6x\geq a \Rightarrow x\geq \frac{a}{6} \\ b-3x\geq 0 \Rightarrow 3x\leq b \Rightarrow x\leq \frac{b}{3} \end{cases} \Rightarrow D_f=\left[\frac{a}{6},\frac{b}{3}\right] \Rightarrow \frac{a}{6}=\frac{b}{3}=2$$

$$\Rightarrow a=12\,,\ b=6$$

$D_f=\{2\}$

$$\frac{b}{a}=? \Rightarrow \frac{6}{12}=\boxed{\frac{1}{2}}$$

---

⑥

الف) $y=\sqrt{\dfrac{x^2-x-6}{x^4-13x^2+36}} \Rightarrow y=\sqrt{\dfrac{(x+2)(x-3)}{(x^2-4)(x^2-9)}}$ (-2, 3 ; ± 2, ± 3)

x		-3		-2^*		2		3^*	
	$+$	$\not 0$	$-$	$\not 0$	$-$	$\not 0$	$+$	$\not 0$	$+$

$$\Rightarrow D_f=(-\infty,-3)\cup(2,3)\cup(3,+\infty)$$

ب) $y=\sqrt{\dfrac{x^3-2x^2-5x+6}{x^3+2x^2-5x-6}}$

$x^3-2x^2-5x+6 \div (x-1) = x^2-x-6 \Rightarrow (x-3)(x+2)(x-1)$ صورت

$x^3+2x^2-5x-6 \div (x+1) = x^2+x-6 \Rightarrow (x+3)(x-2)(x+1)$ مخرج

$$\Rightarrow y=\sqrt{\frac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)}}$$

(1, 3, -2 ; -1, -3, 2)

x		-3		-2		-1		1		2		3	
	$+$	$\not 0$	$-$	0	$+$	$\not 0$	$-$	0	$+$	$\not 0$	$-$	0	$+$

$$\Rightarrow D_f=(-\infty,-3)\cup[-2,-1)\cup[1,2)\cup[3,+\infty)$$

---

⑤

الف) $y=\sqrt{\dfrac{x+1}{|x-3|}}+\sqrt{\dfrac{6-x}{x^2+2x+4}}$

$$\Rightarrow \begin{cases} \dfrac{x+1}{|x-3|}\geq 0 \rightarrow \text{در مخرج } x\neq 3\,,\ x+1\geq 0 \Rightarrow x\geq -1 \\ \dfrac{6-x}{x^2+2x+4}\geq 0 \end{cases}$$

مخرج همیشه مثبت — چون Δ منفی داره — $6-x\geq 0 \Rightarrow x\leq 6$

x		-1		3^*	
	$-$	0	$+$	$\not 0$	$+$

x		6	
	$+$	0	$-$

$$\Rightarrow D_f=[-1,6]-\{3\}$$

ب) $\sqrt{[x]-4}+\sqrt{2-[x]} \Rightarrow \begin{cases} [x]-4\geq 0 \Rightarrow [x]\geq 4 \Rightarrow x\geq 4 \\ 2-[x]\geq 0 \Rightarrow [x]\leq 2 \Rightarrow x<3 \end{cases} \Rightarrow D_f=\varnothing$

(٧)

الف) $y=\sqrt{[-x]-1} \Rightarrow [-x]-1\geq 0 \Rightarrow [-x]\geq 1 \Rightarrow -x\geq 1 \Rightarrow x\leq -1 \Rightarrow x<-1 \Rightarrow D_f=(-\infty,-1)$

ب) $y=\frac{ax^2+1}{[x]^2-3[x]-4} \Rightarrow [x]^2-3[x]-4\neq 0 \Rightarrow ([x]-4)([x]+1)\neq 0 \Rightarrow \begin{cases}[x]\neq 4 \Rightarrow [4,5)\\ [x]\neq -1 \Rightarrow [-1,0)\end{cases}$

$\Rightarrow D_f=\mathbb{R}-\{(-1,0)\cup[4,5)\}$

---

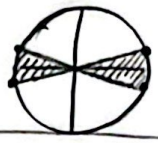
(٨)

الف) $y=\frac{\cot x+1}{\tan x+1} \rightarrow \begin{cases}\tan x+1\neq 0 \Rightarrow \tan x\neq -1 \Rightarrow x\neq k\pi-\frac{\pi}{4}\\ \sin x\neq 0 \Rightarrow x\neq \frac{k\pi}{2}\end{cases}$

$\Rightarrow D_f=\mathbb{R}-\left\{k\pi-\frac{\pi}{4},\frac{k\pi}{2}\right\}$

ب) $y=\sqrt{1-4\sin^2 x} \Rightarrow 1-4\sin^2 x\geq 0 \Rightarrow -4\sin^2 x\geq -1 \Rightarrow \sin^2 x\leq \frac{1}{4} \Rightarrow -\frac{1}{2}\leq \sin x\leq \frac{1}{2}$

$\Rightarrow D_f=\left[k\pi-\frac{\pi}{6},k\pi+\frac{\pi}{6}\right]$



---

(٩)

الف) $y=\sqrt{1-\log_{\frac{1}{2}}^{x-1}} \Rightarrow \begin{cases}x-1>0 \Rightarrow x>1\\ 1-\log_{\frac{1}{2}}^{(x-1)}\geq 0 \Rightarrow \log_{\frac{1}{2}}^{x-1}\leq 1 \Rightarrow x-1\geq \frac{1}{2} \Rightarrow x\geq \frac{3}{2}\end{cases}$

$\Rightarrow D_f=[\frac{3}{2},+\infty)$

ب) $y=\frac{\sqrt{x^2-x}}{1-\log_3^{x^2-3x}} \Rightarrow \begin{cases}x^2-3x>0 \Rightarrow x(x-3)>0 \Rightarrow \overset{x}{\underset{+\ |\ -\ |\ +}{\overset{0}{\ }\ \overset{3}{\ }}}\\ x^2-x\geq 0 \Rightarrow x(x-1)\geq 0 \Rightarrow \overset{x}{\underset{+\ \phi\ -\ \phi\ +}{\overset{0}{\ }\ \overset{1}{\ }}} \Rightarrow (-\infty,0]\cup[1,+\infty)\\ 1-\log_3^{x^2-3x}\neq 0 \Rightarrow \log_3^{x^2-3x}\neq 1 \Rightarrow x^2-3x\neq 3 \Rightarrow x^2-3x-3\neq 0\end{cases}$

$\Rightarrow (x+1)(x-4)\neq 0 \Rightarrow x\neq -1, x\neq 4$

$\Rightarrow D_f=((-\infty,0]\cup[1,+\infty))-\{-1,4\}$

---

(١٠)

الف) $y=\sqrt{4^{6x-x^2}-17} \Rightarrow 4^{6x-x^2}-17\geq 0 \Rightarrow 4^{6x-x^2}\geq 17 \Rightarrow 4^{6x-x^2}\geq 4^2$

$\Rightarrow 6x-x^2\geq 2 \Rightarrow -x^2+6x-2\geq 0 \Rightarrow (-x+1)(x-2)\geq 0 \Rightarrow \begin{cases}x=1\\ x=2\end{cases}$ $\overset{x}{\underset{-\ \phi\ +\ \phi\ -}{\overset{1}{\ }\ \overset{2}{\ }}}$

$\Rightarrow D_f=[1,2]$

ب) $y=\left(\frac{3x+5}{2x+4}\right)! \Rightarrow \frac{3x+5}{2x+4}\in w \Rightarrow 3x+5=2xw+4w \Rightarrow 3x-2xw=4w-5$

$\Rightarrow x=\frac{4w-5}{3-2w} \rightarrow D_f=\left\{x \mid x=\frac{4k-5}{3+2k}, k\in w\right\}$