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التمرين 14

$$x^2 + y^2 - 4x + 4y + k = 0$$

$$x^2 - 4x + y^2 + 4y + k = 0 \rightarrow 2(x-2)^2 - 4 + (y+2)^2 - 4 + k = 0$$

$$2(x-2)^2 + (y+2)^2 + (k-12) = 0 \rightarrow k-12 \leq 0 \rightarrow k \leq 12$$

نقطة المركز $(2, -2)$ $k=12$ نقطة 1, 15

$$a^2 + \varepsilon a = 2a + a + 2 \rightarrow a = 1 \text{ (circled)} \leftarrow a < a$$

$$f(x) = \begin{cases} x^2 + 4x & \text{if } x \geq 1 \\ 2x + 1 + 2 & \text{if } x < 1 \end{cases} \rightarrow \begin{cases} f(1) = 1 + 4 = 5 \\ f(1) = 2 + 3 = 5 \end{cases} \leftarrow f(1) = 5$$

$$|x+1| \geq 2 \rightarrow x+1 \geq 2 \vee x+1 \leq -2 \rightarrow x \geq 1 \vee x \leq -3$$

از اینجا به بعد $x \leq -3$ و دامنه ضابطه دوم عضو است $(-3, 0)$ و اینها را در نظر بگیریم:

$$f(9) - b = a + 2(-3) \rightarrow 18 - b = a - 6 \rightarrow a + b = 24$$

$$y = \sqrt{4x-a} + \sqrt{b-3x}$$

$$4x-a \geq 0 \rightarrow x \geq \frac{a}{4} \quad b-3x \geq 0 \rightarrow x \leq \frac{b}{3}$$

نقطه $\rightarrow \frac{a}{4} = \frac{b}{3} = 2 \rightarrow a=8 \quad b=6$

$$\frac{b}{a} = \frac{6}{8} = \frac{3}{4}$$

$$y = \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+1x+4}} \rightarrow x$$

$$x-3 \neq 0 \rightarrow x \neq 3 \quad x+1 \geq 0 \rightarrow x \geq -1$$

$$x^2 + x + 4 \neq 0 \rightarrow \text{دائماً مثبت و غیر صفر}$$

$$4-x \geq 0 \rightarrow x \leq 4$$

منطقه $D = [-1, 3) \cup (3, 4]$

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ب) $\sqrt{[x]-4} + \sqrt{2-[x]}$

$[x]-4 \geq 0 \rightarrow [x] \geq 4 \rightarrow x \geq 4$
 $2-[x] \geq 0 \rightarrow [x] \leq 2 \rightarrow x < 3$ } $\rightarrow D = \emptyset$

$D = [-\infty, 3) \cup [4, +\infty)$

الف) $\sqrt{\frac{x^2-x-4}{x^4-13x^2+44}} \rightarrow \frac{(x-4)(x+2)}{(x^2-9)(x^2-4)}$

$x^2-9 \neq 0 \rightarrow x^2 \neq 9 \rightarrow x \neq \pm 3$ $x^2-4 \neq 0 \rightarrow x^2 \neq 4 \rightarrow x \neq \pm 2$

-3	-2	2	3
+	-	-	+
تن	تن	تن	تن

$D = (-\infty, 3) \cup (2, +\infty)$

$D = (-\infty, -2) \cup (2, +\infty) - \{x\}$

1, V O

ب) $\sqrt{\frac{x^3-2x^2-5x+4}{x^3+2x^2-5x-4}}$

$x^3-2x^2-5x+4 \xrightarrow{\text{تقسیم بر } x-3} x^2+4x-1 \rightarrow x=3 \rightarrow$ بخش پذیر است

$x^3-2x^2-5x+4 \rightarrow (x-3)(x^2+4x-1) \rightarrow (x-3)(x+2)(x-1)$

$x^3+2x^2-5x-4 \xrightarrow{\text{تقسیم بر } x-2} x^2+4x-1 \rightarrow x=2 \rightarrow$ بخش پذیر است

$x^3+2x^2-5x-4 \rightarrow (x-2)(x^2+4x-1) \rightarrow (x-2)(x+1)(x+3)$

$\frac{(x-3)(x+2)(x-1)}{(x+3)(x-2)(x+1)} \geq 0$

-3	-2	-1	1	2	3
+	-	+	-	+	+
تن	تن	تن	تن	تن	تن

$D = (-\infty, -3) \cup [-2, -1) \cup (1, 2) \cup (3, +\infty)$

الف) $y = \sqrt{[-x]-2} \rightarrow [-x]-2 \geq 0 \rightarrow [-x] \geq 2$

$D \Rightarrow x < -1$ $x < -2$

1, V O

$D = \mathbb{R} - [2, 4) \cup [-1, 0)$

ب) $y = \frac{2x^2+4}{[x]^2-1[x]-1}$

$[x]^2-1[x]-1 \rightarrow ([x]-1)([x]+1)$

$[x]-1 \neq 0 \rightarrow [x] \neq 1 \rightarrow x \notin [1, 2)$ $[x]+1 \neq 0 \rightarrow [x] \neq -1$

$x \notin [-1, 0)$

الف) $y = \frac{\cot x + 1}{\tan x + 1} \rightarrow \tan x = \frac{\pi}{4} + k\pi$ - 1
 $\rightarrow \cot x = k\pi$

$\tan x + 1 \neq 0 \rightarrow \tan x \neq -1 \rightarrow x = -\frac{\pi}{4} + k\pi$

$D = \mathbb{R} - \left\{ \frac{\pi}{4} + k\pi, k\pi, -\frac{\pi}{4} + k\pi \right\}$

$D = \mathbb{R} - \{180^\circ, 270^\circ, 135^\circ, 45^\circ, 315^\circ, 0^\circ\}$

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ب) $\sqrt{1 - \varepsilon \sin^2 x} \rightarrow 1 - \varepsilon \sin^2 x \geq 0 \rightarrow \sin^2 x \leq \frac{1}{\varepsilon}$

$|\sin x| \leq \frac{1}{\sqrt{\varepsilon}} \rightarrow \sin x = \frac{1}{\sqrt{\varepsilon}} \rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}$

$\sin x = -\frac{1}{\sqrt{\varepsilon}} \rightarrow x = \frac{5\pi}{4}, \frac{7\pi}{4}$

$D = \left[k\pi + \frac{\pi}{4}, k\pi + \frac{3\pi}{4} \right] \cup \left[k\pi + \frac{5\pi}{4}, k\pi + \frac{7\pi}{4} \right]$

الف) $y = \sqrt{1 - \log_{\frac{1}{2}}(x-1)}$ 9

$1 - \log_{\frac{1}{2}}(x-1) \geq 0 \rightarrow \log_{\frac{1}{2}}(x-1) \leq 1$

$x-1 > 0 \rightarrow x > 1$
 $\frac{1}{2} \leq x-1 \rightarrow x \geq \frac{3}{2}$
 $D \rightarrow x \geq \frac{3}{2}$

ب) $\frac{\sqrt{x^2 - x}}{1 - \log_{\varepsilon}(x^2 - 3x)} \rightarrow x^2 - x \geq 0 \rightarrow x(x-1) \geq 0$

$1 - \log_{\varepsilon}(x^2 - 3x) \neq 0 \rightarrow \log_{\varepsilon}(x^2 - 3x) \neq 1$
 $x \geq 0 \rightarrow x \geq 1$
 $x \neq -1$

$x^2 - 3x > 0 \rightarrow x(x-3) > 0 \rightarrow x > 3 \rightarrow x-3 > 0 \rightarrow x > 3$

$x \neq x^2 - 3x \rightarrow x(x-3) \neq x \rightarrow x \neq 4$

$D = (-\infty, -1) \cup (-1, 0] \cup [1, 3) \cup (3, +\infty) - \{4\}$

(1, 3, 4)

$$\sqrt{x^2 - 2x} - 1 \rightarrow x^2 - 2x - 1 \geq 0$$

$$x^2 - 2x \geq 1 \rightarrow x^2 - 2x - 1 \geq 0 \rightarrow x^2 - 2x + 1 \geq 1$$

$$D = [0, 2]$$

$$\frac{1}{x} - 2 + x$$

$$\text{ج) } \left(\frac{x+d}{x+\varepsilon} \right)! \quad \frac{x+d}{x+\varepsilon} \in \mathbb{N} \rightarrow \frac{x+d}{x+\varepsilon} \geq 0$$

$$x + \varepsilon \neq 0 \rightarrow x \neq -\varepsilon \rightarrow x \neq -2$$

$$D = \{x \in \mathbb{R} \mid x \neq -2\}$$

$$D = \left\{ x \mid x \in \frac{-x+d}{x-c}, KEW \right\}$$