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نام و نام خانوادگی: ... شماره تألیف ۲ — یازدهم گروه دو دفتر A

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۱) $2x^2+y^2-4x+6y+k=0 \rightarrow 2x^2+y^2-4x+6y+\underset{9+2}{11}=0 \rightarrow$

$(2x^2-4x+2)+(y^2+6y+9)=2(x^2-2x+1)+(y^2+6y+9)=2(x-1)^2+(y+3)^2=0$

$\rightarrow (x-1)=0 \rightarrow x=1 \ , \ y+3=0 \rightarrow y=-3 \rightarrow (1,-3)$

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۲) $f(x)=\begin{cases} x^2+4x & x\geq a \\ 2x+a+2 & x\leq a \end{cases} \rightarrow a^2+4a=2a+a+2 \Rightarrow a^2+a-2=0$

$(a-2)(a+1)=0 \rightarrow \begin{cases} a=2 \\ a=-1 \end{cases}$ غ ق ق — $a=1$

$\rightarrow f(x)=\begin{cases} x^2+4x & x\geq 2 \\ 2x+4 & x\leq 2 \end{cases} \rightarrow f(1)=6$

$f(1)=5$

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۳) $|x+1|\geq 2 \rightarrow \begin{cases} x+1\geq 2 \rightarrow x\geq 1 \\ x+1\leq -2 \rightarrow x\leq -3 \end{cases}$

$f(x)=\begin{cases} 2x^2-b & x\geq 1 \\ 2x^2-b & x\leq -3 \\ a+2x & -3<x\leq -1 \end{cases} \rightarrow f(-3)=18-b=a-6 \Rightarrow 24=a+b$

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۴) $6x-a\geq 0 \rightarrow 6x\geq a \rightarrow x\geq \frac{a}{6}$

$b-3x\geq 0 \rightarrow b\geq 3x \Rightarrow x\leq \frac{b}{3}$

$\rightarrow x=\frac{a}{6}=\frac{b}{3}=2 \rightarrow a=12,\ b=6 \rightarrow \frac{b}{a}=\frac{1}{2}$

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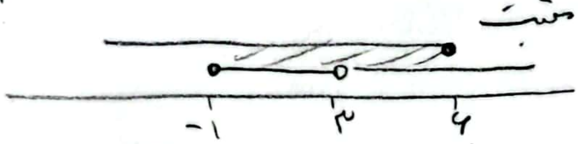
۵) الف) $\sqrt{\frac{x+1}{|x-3|}}\geq 0+\sqrt{\frac{6-x}{x^2+2x+4}}\geq 0$

$|x-3|>0$

$x+1\geq 0 \rightarrow x\geq -1$

$[-1,+\infty)-\{3\}$

$\frac{6-x}{x^2+2x+4}\geq 0 \rightarrow \Delta=-12<0$ مخرج همواره مثبت $\Rightarrow 6-x\geq 0 \rightarrow x\leq 6$



$D=[-1,6]-\{3\}$

ب) $y=\sqrt{[x]-4}+\sqrt{2-[x]}$

$[x]-4\geq 0 \rightarrow [x]\geq 4 \rightarrow x\geq 4 \rightarrow [4,+\infty)$

$2-[x]\geq 0 \rightarrow [x]\leq 2 \rightarrow x<3 \rightarrow (-\infty,3)$

$\rightarrow D_f=[4,+\infty)\cap(-\infty,3)=\varnothing$

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۶) الف) $y=\sqrt{\frac{x^2-x-6}{x^4-13x^2+36}}\geq 0$

$x^2-x-6=0 \rightarrow (x-3)(x+2)=0 \rightarrow \begin{cases} x=3 \\ x=-2 \end{cases}$

$x^4-13x^2+36=(x^2-9)(x^2-4)=0 \rightarrow \begin{cases} x=\pm 3 \\ x=\pm 2 \end{cases}$

	-3		-2		2		3	
$-$		$+$		$-$		$+$		$-$

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ب) $y=\sqrt{\frac{x^3-2x^2-5x+4}{x^3+2x^2-5x-4}}\geq 0$

$x^3-2x^2-5x+4=0 \xrightarrow{\text{تجزیه}} (x-1)(x^2-x-4)=(x-1)(x-4)(x+2)=0$

$x^3-2x^2-5x+4 \;\big|\; x-1$

x^3-x^2

$-x^2-5x$

$-x^2+x$

$-4x+4$

$-4x+4$

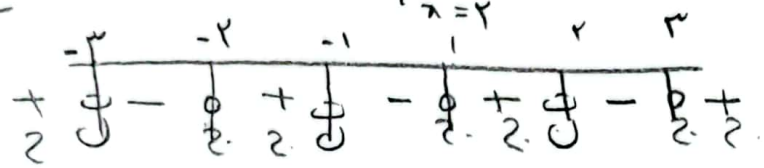
0

x^2-x-4

$\rightarrow \begin{cases} x=1 \\ x=4 \\ x=-2 \end{cases}$

$x^3+2x^2-5x-4=0 \rightarrow (x+1)(x^2+x-4)=(x+1)(x+4)(x-2)=0$

$\begin{cases} x=-1 \\ x=-4 \\ x=2 \end{cases}$



$(-\infty,-4)\cup[-2,-1)\cup[1,2)\cup[4,+\infty)$

(ب) $y=\sqrt{[-x]-2}\geq 0 \rightarrow [-x]\geq 2 \longrightarrow -x\geq 2 \longrightarrow x\leq -2 \rightarrow (-\infty,-2]$

(ب) $y=\frac{5x^2+4}{[x]^2+[x]-2} \rightarrow [x]=t \rightarrow t^2+t-2=0 \rightarrow (t-1)(t+2)=0$

$\begin{cases} t=1 \longrightarrow [x]=1 \longrightarrow 1\leq x<2 \longrightarrow [1,2) \\ t=-2 \rightarrow [x]=-2 \longrightarrow -2\leq x<-1 \rightarrow [-2,-1) \end{cases} \longrightarrow D=\mathbb{R}-([-2,-1)\cup[1,2))$

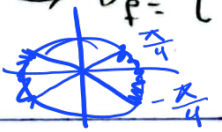
ج) $y=\frac{\cot x+1}{\tan x+1}$ $\quad \tan x+1\neq 0 \rightarrow \tan x\neq -1 \rightarrow x\neq k\pi-\frac{\pi}{4}$

$\cos x\neq 0$ $\quad \sin x\neq 0 \rightarrow x\neq k\pi$

$D=R-\{k\pi, k\pi-\frac{\pi}{4}\}$

ب) $y=\sqrt{1-\frac{1}{4}\sin^2 x}\geq 0 \rightarrow \frac{1}{4}\geq \frac{1}{4}\sin^2 x \rightarrow \sin^2 x\leq 1 \rightarrow |\sin x|\leq 1$

$-\frac{1}{2}\leq \sin x\leq \frac{1}{2}$ $\quad \rightarrow D_f=[2k\pi+\frac{\pi}{4}, 2k\pi+\frac{5\pi}{4}]$



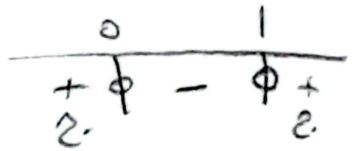
د) $y=\sqrt{1-\log_{\frac{1}{2}}^{x-1}}\geq 0$

$\begin{cases} 1-\log_{\frac{1}{2}}^{x-1}\geq 0 \rightarrow \log_{\frac{1}{2}}^{x-1}\leq 1 \longrightarrow x-1\geq \frac{1}{2} \rightarrow x\geq \frac{3}{2} \\ x-1>0 \rightarrow x>1 \\ \frac{1}{2}>0 \checkmark \\ \frac{1}{2}\neq 1 \checkmark \end{cases}$

$D=(1,\frac{3}{2}]$

ب) $y = \frac{\sqrt{x^2 - x}}{1 - \log_4^{x^2 - 3x}}$

$x^2 - x \geq 0 \longrightarrow x(x-1) = 0 \longrightarrow \begin{cases} x = 0 \\ x = 1 \end{cases}$

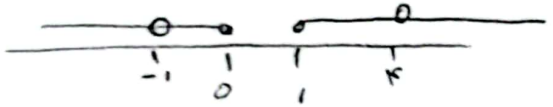


$\textcircled{1}\ (-\infty, 0] \cup [1, +\infty)$

$1 - \log_4^{x^2 - 3x} \neq 0$

$1 = \log_4^{x^2 - 3x} \Rightarrow x^2 - 3x \neq 4 \longrightarrow x^2 - 3x - 4 \neq 0$

$\longrightarrow (x-4)(x+1) \neq 0 \quad x \neq 4,\ x \neq -1 \quad \textcircled{2}$



$D_f = (-\infty, 0] \cup [1, +\infty) - \{-1, 4\}$

الف) $f = \sqrt{3^{4x - x^2} - 1} \geq 0 \qquad 3^{4x - x^2} \geq 1 = 3^0$

$-x^2 + 4x \geq 0 \longrightarrow -(-x^2 + 4x - 3) \geq 0 \qquad x^2 - 4x + 3 \leq 0 \qquad (x-3)(x-1)$

$x = 3 \qquad x = 1$

$D_f = (-\infty, 1] \cup [3, +\infty)$

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ب) $\left(\frac{3x + 5}{2x + 4}\right)!$ ✓

$\frac{3x + 5}{2x + 1} \in \omega \longrightarrow x = -\frac{4\omega - 5}{2\omega - 3}$

$D = \{x \mid x = -\frac{4k + 5}{2k - 3},\ k \in \omega\}$