نام و نام خانوادگی = ویانا کاسه‌ای   شماره تألیف = ۲   یازدهم گروه ۵۰۰ دفتر A

۱)

$2x^2+y^2-4x+6y+k=0 \rightarrow 2x^2+y^2-4x+6y+\underset{9+2}{11}=0 \rightarrow$

$(2x^2-4x+2)+(y^2+6y+9)=2(x^2-2x+1)+(y^2+6y+9)=2(x-1)^2+(y+3)^2=0$

$\rightarrow (x-1)=0 \rightarrow x=1 \ , \ y+3=0 \rightarrow y=-3 \rightarrow (1,-3)$

۲)

$f(x)=\begin{cases} x^2+4x & x\geq a \\ 2x+a+2 & x\leq a \end{cases} \rightarrow a^2+4a=2a+a+2 \Rightarrow a^2-a-2=0$

$(a-2)(a+1)=0 \rightarrow \begin{cases} a=2 \\ a=-1 \end{cases}$ غ ق ق

$\rightarrow f(x)=\begin{cases} x^2+4x & x\geq 2 \\ 2x+4 & x\leq 2 \end{cases} \rightarrow f(1)=6$

۳)

$|x+1|\geq 2 \rightarrow \begin{cases} x+1\geq 2 \rightarrow x\geq 1 \\ x+1\leq -2 \rightarrow x\leq -3 \end{cases}$

$f(x)=\begin{cases} 2x^2-b & x\geq 1 \\ 2x^2-b & x\leq -3 \\ a+2x & -3<x<1 \end{cases} \rightarrow f(-3)=18-b=a-6 \Rightarrow \boxed{24=a+b}$

۴)

$6x-a\geq 0 \rightarrow 6x\geq a \rightarrow x\geq \frac{a}{6}$

$b-3x\geq 0 \rightarrow b\geq 3x \Rightarrow x\leq \frac{b}{3}$

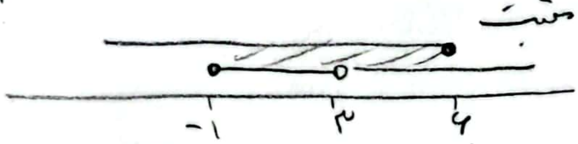
$\rightarrow x=\frac{a}{6}=\frac{b}{3}=2 \rightarrow \begin{matrix} a=12 \\ b=6 \end{matrix} \rightarrow \frac{b}{a}=\frac{1}{2}$

۵)

الف) $\sqrt{\frac{x+1}{\underbrace{|x-3|}_{\geq 0}}}\geq 0 + \sqrt{\frac{6-x}{x^2+2x+4}}\geq 0$

$\Downarrow$ $x+1\geq 0 \rightarrow x\geq -1$ ، $[-1,+\infty)-\{3\}$

$\Downarrow$ $\frac{6-x}{x^2+2x+4}\geq 0 \rightarrow \Delta=-12<0$ مخرج همواره مثبت $\Rightarrow 6-x\geq 0 \rightarrow x\leq 6$



$D=[-1,6]-\{3\}$

ب) $y=\sqrt{[x]-4}+\sqrt{2-[x]}$

$[x]-4\geq 0 \rightarrow [x]\geq 4 \rightarrow x\geq 4 \rightarrow [4,+\infty)$

$2-[x]\geq 0 \rightarrow [x]\leq 2 \rightarrow x<3 \rightarrow (-\infty,3)$

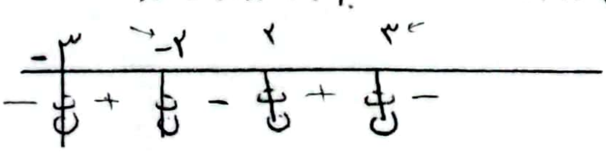
$\rightarrow D_f=[4,+\infty)\cap(-\infty,3)=\varnothing$

۶)

الف) $y=\sqrt{\frac{x^2-x-6}{x^4-13x^2+36}}\geq 0$

$x^2-x-6=0 \rightarrow (x-3)(x+2)=0 \rightarrow \begin{cases} x=3 \\ x=-2 \end{cases}$

$x^4-13x^2+36=(x^2-9)(x^2-4)=0 \rightarrow \begin{cases} x=\pm 3 \\ x=\pm 2 \end{cases}$



ب) $y=\sqrt{\frac{x^3-2x^2-5x+4}{x^3+2x^2-5x-4}} \geq 0$

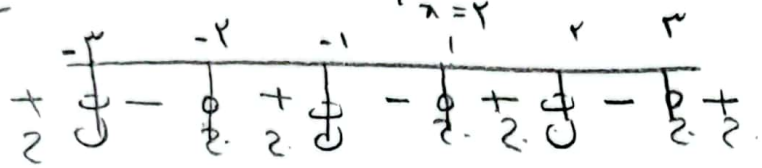
$x^3-2x^2-5x+4=0 \xrightarrow{\text{تجزیه}} (x-1)(x^2-x-4)=(x-1)(x-4)(x+2)=0$

$\begin{array}{r|l} x^3-2x^2-5x+4 & x-1 \\ \hline -x^3+x^2 & x^2-x-4 \\ \hline -x^2-5x & \\ -x^2+x & \\ \hline -4x+4 & \\ -4x+4 & \\ \hline 0 & \end{array}$

$\rightarrow \begin{cases} x=1 \\ x=4 \\ x=-2 \end{cases}$

$x^3+2x^2-5x-4=0 \rightarrow (x+1)(x^2+x-4)=(x+1)(x+4)(x-2)=0$

$\begin{cases} x=-1 \\ x=-4 \\ x=2 \end{cases}$



$(-\infty,-4) \cup [-2,-1) \cup [1,2) \cup [4,+\infty)$

(ت) $y=\sqrt{[-x]-2} \geq 0 \rightarrow [-x] \geq 2 \longrightarrow -x \geq 2 \longrightarrow x \leq -2 \rightarrow (-\infty,-2]$

ب) $y=\frac{5x^2+4}{[x]^2+[x]-2} \rightarrow [x]=t \rightarrow t^2+t-2=0 \rightarrow (t-1)(t+2)=0$

$\begin{cases} t=1 \longrightarrow [x]=1 \longrightarrow 1 \leq x < 2 \longrightarrow [1,2) \\ t=-2 \rightarrow [x]=-2 \longrightarrow -2 \leq x < -1 \rightarrow [-2,-1) \end{cases} \longrightarrow D=R-([-2,-1) \cup [1,2))$

ج) $y=\frac{\cot x+1}{\tan x+1}$ $\quad \tan x+1 \neq 0 \rightarrow \tan x \neq -1 \rightarrow x \neq k\pi-\frac{\pi}{4}$

$\sin x \neq 0 \rightarrow x \neq k\pi$

$D=R-\{k\pi, k\pi-\frac{\pi}{4}\}$

ت) $y=\sqrt{1-\frac{1}{4}\sin^2 x} \geq 0 \rightarrow \frac{1}{4} \geq \frac{1}{4}\sin^2 x \rightarrow \sqrt{\sin^2 x} \leq \sqrt{\frac{1}{4}} \rightarrow |\sin x| \leq \frac{1}{2}$

$-\frac{1}{2} \leq \sin x \leq \frac{1}{2} \rightarrow D_f=[2k\pi+\frac{\pi}{4}, 2k\pi+\frac{5\pi}{4}]$

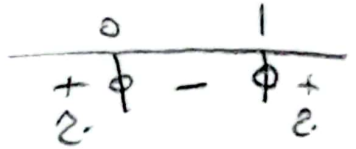
ح) $y=\sqrt{1-\log_{\frac{1}{2}}^{x-1}} \geq 0$

$\begin{cases} 1-\log_{\frac{1}{2}}^{x-1} \geq 0 \rightarrow \log_{\frac{1}{2}}^{x-1} \leq 1 \longrightarrow x-1 \geq \frac{1}{2} \rightarrow x \geq \frac{3}{2} \\ x-1>0 \rightarrow x>1 \\ \frac{1}{2}>0 \checkmark \\ \frac{1}{2} \neq 1 \checkmark \end{cases}$

$D=(1,\frac{3}{2}]$

ب) $y = \dfrac{\sqrt{x^2 - x}}{1 - \log_4^{x^2-3x}}$

$x^2 - x \geq 0 \longrightarrow x(x-1) = 0 \longrightarrow \begin{cases} x = 0 \\ x = 1 \end{cases}$

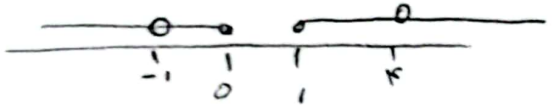


$(-\infty, 0] \cup [1, +\infty)$ ①

$1 - \log_4^{x^2-3x} \neq 0$

$1 = \log_4^{x^2-3x} \Rightarrow x^2 - 3x \neq 4 \longrightarrow x^2 - 3x - 4 \neq 0$

$\longrightarrow (x-4)(x+1) \neq 0 \quad x \neq 4, x \neq -1$ ②



$D_f = (-\infty, 0] \cup [1, +\infty) - \{-1, 4\}$

الف) $f = \sqrt{3^{4x - x^2} - 1} \geq 0 \qquad 3^{4x - x^2} \geq 1 = 3^0$

$-x^2 + 4x \geq 0 \longrightarrow -(-x^2 + 4x - 3) \geq 0 \qquad x^2 - 4x + 3 \leq 0 \qquad (x-3)(x-1)$

$x = 3 \quad x = 1$

$D_f = (-\infty, 1] \cup [3, +\infty)$

ب) $\left(\dfrac{3x + \omega}{2x + 4}\right)!$ $\qquad \dfrac{3x + \omega}{2x + 4} \in \omega \longrightarrow x = \dfrac{-4\omega - \omega}{2\omega - 3}$

$D = \left\{ x \mid x = \dfrac{-4k + \omega}{2k - 3}, k \in \omega \right\}$