

$$x-1 = \frac{-1}{n-1} - (n-1)^r = -1 \quad \text{نقد}$$

$$x-1 = \frac{1}{n-1} \rightarrow (n-1)^r = 1 \xrightarrow{r} |n-1|=1 \rightarrow n=2 \quad \begin{matrix} \text{نقد} \\ \text{نقد} \end{matrix}$$

$$1) \quad -r < \frac{r(n-1)}{n} < r \xrightarrow{\text{در دو طرف ضرب}} -rn < r(n-1) < rn \rightarrow -rn < rn-1$$

$$2) \quad \left| \frac{n-r}{r(n+1)} \right| > 1 \rightarrow \frac{n-r}{r(n+1)} > 1 \rightarrow \frac{n-r}{r(n+1)} > -\frac{r}{r(n+1)} = -\frac{1}{n+1}$$

$$|n-\alpha| < \beta \rightarrow -\beta < n-\alpha < \beta$$

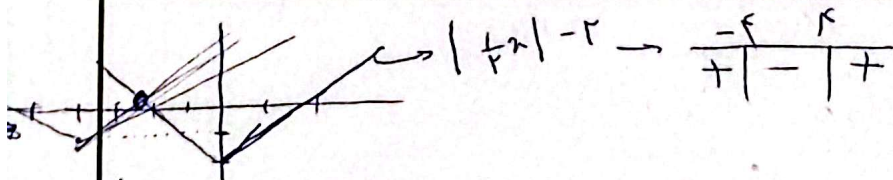
$$\begin{aligned} x^r < x+y &\rightarrow n^r - n - y < 0 \rightarrow -r < n < r \\ -x^r > x+y &\rightarrow x^r + x + y < 0 \end{aligned}$$

$$x = -1 \rightarrow -r + r = 1$$

$$x = 0 \rightarrow 1 + r = \boxed{r} \text{ max}$$

$$x = 1 \rightarrow r - r = -1 \text{ min}$$

$$\min + \max = \boxed{r}$$



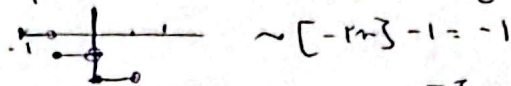
$$\hookrightarrow \left| \frac{1}{r}x + r \right| - 1 \rightarrow \frac{-1}{r} - \frac{-r}{r} = \frac{-1 + r}{r}$$

$$\lfloor |1-x| - |x+1| \rfloor \sim \frac{-x}{1-\Delta |x+1| - \Delta}$$

$$x \sim x \in [-\Delta, \Delta], x \in \mathbb{Z} \rightarrow -\Delta, -\Delta-1, -\Delta-2, \dots, -1, 0, 1, 2, \dots, \Delta-1, \Delta$$

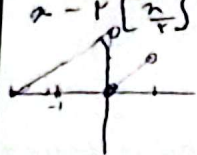
$$b) f(x) = \frac{1}{x} - \left\lfloor \frac{x+1}{1+x} \right\rfloor = \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor - 1, \quad \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor < 1 \Rightarrow -1 \leq \frac{1}{x} - \left\lfloor \frac{1}{x} \right\rfloor - 1 < 0$$

$$a) y = \lfloor -x \rfloor - 1 \quad \begin{cases} -\frac{1}{2} \leq x < 0 \xrightarrow{x \in \mathbb{Z}} \lfloor -x \rfloor = 0 \\ x \in (-1, -\frac{1}{2}) \xrightarrow{x \in \mathbb{Z}} \lfloor -x \rfloor = -1 \end{cases}$$



$$x - \lfloor \frac{x}{2} \rfloor \quad x \in (-1, 1)$$

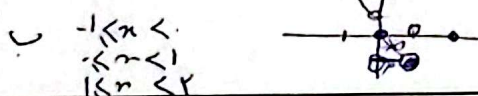
$$\begin{cases} -1 \leq x < -\frac{1}{2} \xrightarrow{x \in \mathbb{Z}} \lfloor -x \rfloor = -1 \\ -\frac{1}{2} \leq x < 0 \xrightarrow{x \in \mathbb{Z}} \lfloor -x \rfloor = 0 \\ 0 \leq x < \frac{1}{2} \xrightarrow{x \in \mathbb{Z}} \lfloor -x \rfloor = -1 \\ \frac{1}{2} \leq x < 1 \xrightarrow{x \in \mathbb{Z}} \lfloor -x \rfloor = -2 \end{cases}$$



$$a) \lfloor |x-1| \rfloor \xrightarrow{-1 \leq x < 0} \lfloor |x-1| \rfloor \sim x \in [-1, 0] \rightarrow \lfloor x-1 \rfloor = 1$$

$$x \in [-1, 0] \rightarrow \lfloor x+1 \rfloor = 0$$

$$\begin{cases} 0 \leq x < 1 \xrightarrow{x \in \mathbb{Z}} \lfloor |x-1| \rfloor = 0 \\ 1 \leq x < 2 \xrightarrow{x \in \mathbb{Z}} \lfloor |x-1| \rfloor = 1 \end{cases}$$



$$\begin{cases} a + \lfloor b \rfloor = 1, 2 \\ b - \lfloor a \rfloor = 1, 2 \end{cases} \sim \lfloor b \rfloor - \lfloor a \rfloor = 1$$

$$a + b + \underbrace{\lfloor b \rfloor - \lfloor a \rfloor}_1 = 1, 4 \rightarrow a + b = 1, 4$$

$$(1+\sqrt{2})^9 + (1-\sqrt{2})^9 = 19 \rightarrow (1+\sqrt{2})^9 = 19 - (1-\sqrt{2})^9$$

$$\lfloor (1+\sqrt{2})^9 \rfloor = \underbrace{(19 - (1-\sqrt{2})^9)}_{19}$$