

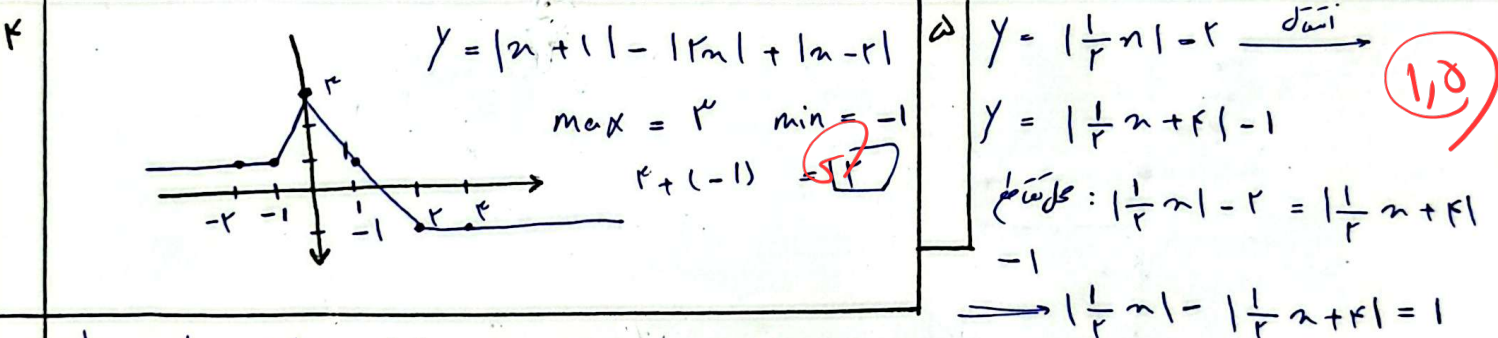
۱۷/۱۵

۱) تنها اعداد معکوسه مانند $\frac{2}{3}$ و $\frac{3}{2}$ و عدد ۱ $\Rightarrow (n-1)(|n-1|) = 1 \Rightarrow \frac{1}{n-1} = |n-1| \Rightarrow$ ضرب خودشان در عدد معکوسشان ۱ می شود، اما هیچگاه این دو عدد معکوسه در این برابری به وجود نمی آید پس تنها:

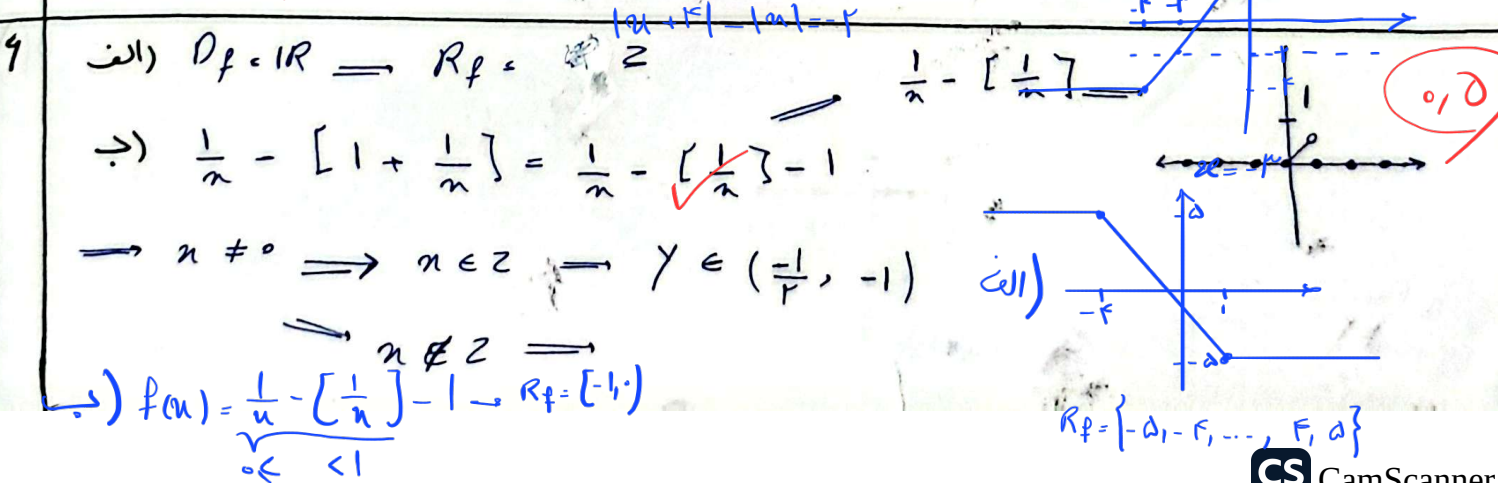
$$n-1 = 1 \Rightarrow n = 2$$

۲) الف) $-2 < \frac{2n-1}{n} < 2 \Rightarrow \frac{2n-1}{n} < 2 : \frac{2n-1-2n}{n} < 0 \Rightarrow \frac{-1}{n} < 0 \Rightarrow \frac{1}{n} > 0 \Rightarrow n > 0$ I
 $I \cap II \in (\frac{1}{4}, +\infty) \Rightarrow \frac{2n-1}{n} > -2 \Rightarrow \frac{2n-1+2n}{n} > 0 \Rightarrow \frac{4n-1}{n} > 0 \Rightarrow \frac{4}{n} > 1 \Rightarrow n < 4$ II
 $\Rightarrow \frac{n-2}{n+1} > 1 \cup \frac{n-2}{n+1} < -1 \Rightarrow \frac{n-2-n-1}{n+1} > 0 \Rightarrow \frac{-2n-3}{n+1} > 0 \Rightarrow \frac{-2}{-1} > \frac{-3}{-1} \Rightarrow -2 < -3$ (contradiction)
 $\Rightarrow \frac{n-2}{n+1} < -1 \Rightarrow \frac{n-2+n+1}{n+1} < 0 \Rightarrow \frac{2n-1}{n+1} < 0 \Rightarrow \frac{-1}{n+1} < 0 \Rightarrow n+1 > 0 \Rightarrow n > -1$
 $\Rightarrow I \cup II \in (-1, \frac{1}{4})$

۳) $n+4 > n^2 \Rightarrow n^2-n-4 < 0 \Rightarrow (n-2)(n+2) < 0 \Rightarrow -2 < n < 2$
 $n+4 < -n^2 \Rightarrow n^2+n+4 < 0 \Rightarrow \emptyset$
 $|x-\alpha| < \beta \Rightarrow \alpha-\beta < x < \alpha+\beta \Rightarrow \alpha = \frac{1}{4}, \beta = \frac{5}{4} \Rightarrow \boxed{a < \frac{1}{4}}$



$\frac{1}{r}n = t \Rightarrow |t| - |t+1| = 1 \Rightarrow |\frac{1}{r}n| - |\frac{1}{r}n + 1| - 1 = 0$
 $|\frac{1}{r}n| - 1 = |\frac{1}{r}n + 1| \Rightarrow \frac{1}{r^2}n^2 + 1 - |n| = \frac{1}{r^2}n^2 + 1 + 2 + n \Rightarrow -|n| = n + 2$
 $|n| = -n - 2 \Rightarrow n^2 = 19n^2 + 12n + 228 \Rightarrow n^2 + 18n + 119 = 0$
 $y = |\frac{1}{r}n + 1| - 1 = y$
 $|n| = -n - 2 \Rightarrow n \leq -2$



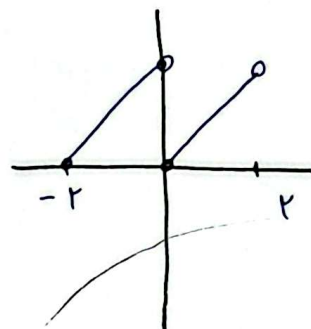
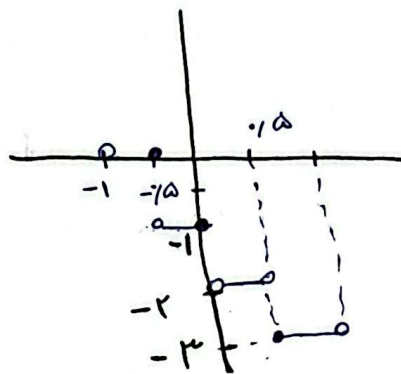
الف) $y = [-2x] - 1 \quad x \in (-1, 1)$

$x \Rightarrow -1 < x < -0.5 \Rightarrow 0$

$-0.5 < x < 0 \Rightarrow -1$

$0 < x < 0.5 \Rightarrow -2$

$0.5 < x < 1 \Rightarrow -3$



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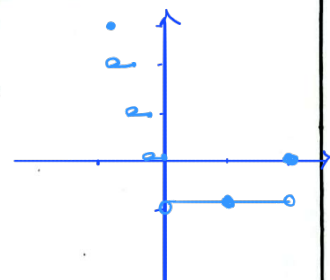
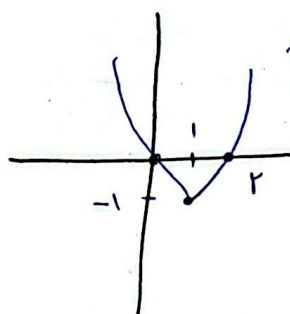
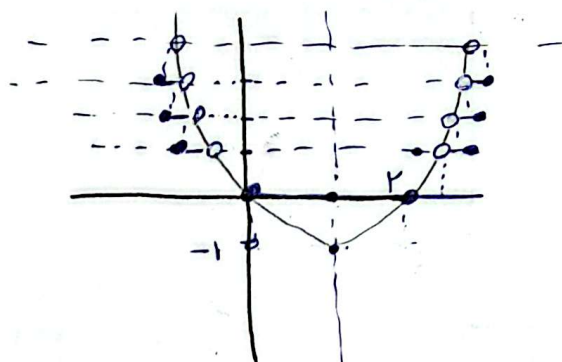
$\Rightarrow x \in [-\frac{r}{r}] \quad x \in (-r, r)$

$a + [b] = 5, 2 \Rightarrow [b] = 5, 2 - a \quad a = k + 0, 2 \quad (a \in [b])$

$b - [a] = 5, 2 \quad b - [k + 0, 2] = 5, 2 \Rightarrow b - k = 5, 2 \quad -k = 5, 2 - b$

$[b] = 5, 2 - k - 0, 2 = 5 - k = 5, 2 + 0, 2 \quad a = 1, 2 \quad b = 5, 2 \quad a + b = 6, 4$

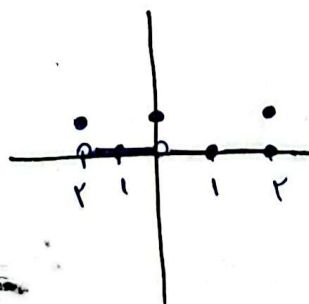
$\Rightarrow y = [x^2 - 2x] \quad x \in [-1, 2]$



الف) $x = -2 \Rightarrow y = 1 \quad -2 < x < -1 \Rightarrow y = 0$

$-1 < x < 0 \Rightarrow y = 0 \quad x = 0 \Rightarrow y = 1 \quad 0 < x < 1 \Rightarrow y = 0$

$y \Rightarrow 1 < x < 2 \Rightarrow y = 0 \quad x = 2 \Rightarrow y = 1$



1, 1, 0, 0

-1.

$$((1+\sqrt{2})^2)^4 + ((1-\sqrt{2})^2)^4 = 191$$

$$(1+2+2\sqrt{2})^4 + (1-2-2\sqrt{2})^4 = 191$$

$$(3+2\sqrt{2}+3-2\sqrt{2})^4 - \left[3 \times 4 \times \cancel{(3-2\sqrt{2})}^1 (3+2\sqrt{2}) \right]$$

$$9^4 - 12 = 191$$

$$(1+\sqrt{2})^4 = (3+2\sqrt{2})^4 = 81 + 192\sqrt{2} + 9\cancel{(1)}^{V2} + 8\sqrt{2}$$
$$= 99 + 192\sqrt{2} + 8\sqrt{2}$$

$$[99 + 192\sqrt{2} + 8\sqrt{2}] = 99 + [2(100\sqrt{2} + 4\sqrt{2})]$$

$$= 191$$